PROBLEM 6.1. Consider the complex amplitude of a wave

\[ U(r) = A \exp(-jK \cdot r), \]

in which \( A \) is the complex envelope, \( K \) is the wave vector (or direction of propagation vector), and \( r \) is the position vector. Demonstrate that the magnitude of vector \( K \) is equal to the wave number \( (k = \frac{2\pi}{\lambda}) \), so that \( U(r) \) is an appropriate wave function representation.

PROBLEM 6.2. Refer to Problem 6.1. Demonstrate that the complex amplitude, as shown in Problem 6.1, represents a plane wave and that its wavefronts are parallel planes perpendicular to the direction of propagation vector \( K \).

PROBLEM 6.3. Demonstrate that the spherical wave characterized by the complex amplitude

\[ U(r) = \frac{A}{r} \exp(-jkr), \]

where \( r \) is the position vector in spherical coordinates and \( k \) is the wave number, satisfies the Helmholtz equation, and therefore, it is a valid wave function representation. Note that the use of spherical coordinates can simplify the demonstration. Also, demonstrate that \( U(r) \) propagates with spherical wavefronts.