General information

**Instructor:** Cosme Furlong  
HL-151  
(508) 831-5126  
cfurlong@wpi.edu  
http://www.wpi.edu/~cfurlong/es2502.html

**Teaching Assistants:** Morteza Khaleghi  
HL-150  
(508) 831-5125  
mkhaleghi@wpi.edu

Tatiana Popova  
tpopova@wpi.edu
Summary
Average normal stress in an axially loaded bar

Figure: 02-01-A-UN
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Figure: 02-01-B-UN
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.
Average normal stress in an axially loaded bar

Internal distribution of forces

\[ \Delta F = \sigma \Delta A \]

\[ \sigma = \frac{P}{A} \]

\[ \int dF = \int \sigma \, dA \]

\[ P = \sigma A \]

\[ \sum F_z = \sum F_z \]
Strain: definition: change in length per unit length

**Normal strain**

**Average normal strain:**
\[
\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}
\]

**Normal strain:**
\[
\varepsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}
\]
**Strain:** definition: change in length per unit length

**Shear strain**

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{B \to A \text{ along } n} \lim_{C \to A \text{ along } t} \theta'$$
Stress ↔ Strain: Hook’s Law

\[ \sigma = E \cdot \varepsilon \]

\( E \) = Elastic modulus (aka)

Remember: \( E \) is nearly the same for different classes of steels!!
Poisson’s ratio:

\[ \nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{longitudinal}}} \]
**Average direct shear stress**

\[ \tau_{avg} = \frac{V}{A_V} \]

Bar subjected to shear load

\( A_{BCD} \) section

(Area under shear)

(Internal loading)
Shear stress ↔ strain

Pure shear

Hook's law for shear: $\tau = G \gamma$

with $G = \frac{E}{2(1 + \nu)}$ (shear modulus)
Statically indeterminate axially loaded member

Axially loaded member

Additional equations are obtained by applying:

*Compatibility or kinematic equations*

\[ \delta_{AB} = 0 \]
Statically indeterminate axially loaded member

Compatibility or kinematic equations:

\[ \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0 \]
Thermal stresses: uniaxial effects

$$\varepsilon_T = \alpha \Delta T$$  \hspace{1cm} $$\delta_T = \varepsilon_T \cdot L = \alpha \Delta T \cdot L$$

(Thermal strains)  \hspace{1cm} (Thermal deformations)

$$\alpha$$ = linear coefficient of thermal expansion, 1/°C, 1/°F
$$\Delta T$$ = temperature differential
$$L$$ = original length of component
According to Hook’s law (linear elasticity):

\[ \tau = G \cdot \gamma \]
The flexure formula

Resultant internal moment:

\[ M = \frac{\sigma_x^{Max}}{c} \int_A y^2 \, dA \]

\[ \sigma_x = \frac{M}{I_{zz}} \frac{y}{y} \]

\[ I_{zz} = \int_A y^2 \, dA \]

Area moment of inertia wrt to z-axis
Shear formula

Observed in components subjected to bending loads

\[ \tau(x, y) = \frac{V(x) \cdot Q(y)}{I_{zz}(x) \cdot t(y)} \]

Important to remember!!

\[ Q = \bar{y}'A' \]

Internal distribution of shear stresses:

\[ \tau_{xy} = \tau_{yx} \]
Shear and bending diagrams: regions with distributed load

\[ \frac{dV}{dx} = w(x) \]

\[ \frac{dM}{dx} = V(x) \]

Important to remember!!

Free body diagram of element \( \Delta x \):

\[ \frac{dV}{dx} = w(x) \]

\[ \frac{dM}{dx} = V(x) \]
Bending deformation of straight beams

The elastic curve

\[
\frac{w}{EI} = \frac{d^4 y}{dx^4}
\]

Load function – deflection

\[
\frac{V}{EI} = \frac{d^3 y}{dx^3}
\]

Shear function – deflection

\[
\frac{M}{EI} = \frac{d^2 y}{dx^2}
\]

Moment function – *elastica*

\[
\theta = \frac{dy}{dx}
\]

Slope – deflection

\[
y = f(x)
\]

Deflection
Bending deformation of straight beams: example A

The cantilever shown is subjected to a vertical load $P$ at its end. Determine the equation of the deformation (elastic) curve. $E \cdot I$ is constant.
Bending deformation of straight beams

The elastic curve

For small deformations:

$$\frac{d^2 y}{dx^2} = \frac{M}{E \cdot I_{zz}}$$

Elastica equation

Important to remember!!
Find the most highly stressed locations on the bracket shown. Draw volume (stress) elements at points A and B.
Plane stress transformation (rotation)

Stress cube in 2D

Rotate cube in 2D while keeping resultant forces the same
Mohr’s circle (developed by Otto Mohr in 1882)

Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- **Principal stresses**:
  - \(\sigma_1\) (Principal stress)
  - \(\sigma_2\) (Principal stress)

- **Applied normal stress**:
  - \(\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}\)

- **Applied shear stress**:
  - \(\tau_{xy}\)

- **Maximum shear**:
  - \(\tau_{\text{max \ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\)
Thank you!!!