Lecture 22:
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General information

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The flexure formula

\[ \sigma_x = \left( \frac{y}{c} \right) \cdot \sigma_{x,\text{Max}} \]

Normal stress variation as a function of \( y \) as predicted by flexure formula

Differential stress element
The flexure formula

Resultant internal moment:

\[ M = \sum_{z} M_z \]

\[ M = \int_{A} y \, dF = \int_{A} y \left( \sigma_x \, dA \right) \]
The flexure formula

Resultant internal moment:

\[ M = \int_A y \, dF = \int_A y \left( \sigma_x \, dA \right) \]

\[ = \int_A y \left( \frac{y}{c} \sigma_{x_{\text{Max}}} \, dA \right) \]
The flexure formula

Resultant internal moment:

\[ M = \frac{\sigma_{x,\text{Max}}}{c} \int_A y^2 \, dA \]

\[ \sigma_{x,\text{Max}} = \frac{M \cdot c}{I_{zz}} \]

\[ I_{zz} = \int_A y^2 \, dA \]

Area moment of inertia wrt to z-axis
The flexure formula

\[ \sigma_{x_{\text{Max}}} = \frac{M \cdot c}{I_{zz}} \]

\[ \sigma_x = -\frac{M \cdot y}{I_{zz}} \]

Do note that:
(we need moment diagram)

\[ \sigma_x(x, y) = -\frac{M(x) \cdot y}{I_{zz}(x)} \]

Important to remember!!
Shear and bending diagrams: example C

A member having the dimensions shown is used to resist an internal bending moment of \( M = 90 \text{ kN} \cdot \text{m} \). Determine the maximum stress in the member if the moment is applied (a) about the \( z \)-axis (as shown); and (b) about the \( y \)-axis. Sketch the stress distribution for each case.
Shear and bending diagrams: example C

Area moment of inertia

(aka, 2\textsuperscript{nd} area moment of inertia):

\[ I_{xx} = \int_A y^2 \, dA , \]
\[ I_{yy} = \int_A x^2 \, dA \]

Polar area moment of inertia:

\[ J_O = I_{xx} + I_{yy} \]
Shear and bending diagrams: example C

Area moment of inertia: \textit{parallel-axis theorem}:

\[ I_{xx} = \bar{I}_{x'x'} + A \cdot d_y^2, \]
\[ I_{yy} = \bar{I}_{y'y'} + A \cdot d_x^2 \]

Polar area moment of inertia: \textit{parallel-axis theorem}:

\[ J_o = \bar{J}_C + A \cdot d^2 \]
Shear and bending diagrams: example C

\[ I_{xx} = \frac{1}{12} b \cdot h^3, \quad I_{yy} = \frac{1}{12} h \cdot b^3 \]
Shear and bending diagrams: example D

A box beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross-section is 10 kN⋅m, determine the stresses at points $A$ and $B$ and show the results acting on volume elements located at these points.

$M = 10 \text{kN} \cdot \text{m}$

**Note:** state of stresses is *independent* of material (properties)
Shear and bending diagrams: example E

The axle of the freight car is subjected to wheel loadings of 20 kip. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 5.5 in. **Apply two different methods**
Stress concentrations: normal stresses: bending

Fringe pattern obtained with photoelasticity: pattern reveals distribution of internal stresses
Stress concentrations: bending: normal stresses + shear

**Bending**

Nominal bending stress:
\[
\sigma_{\text{nom}} = \frac{M_d}{I}
\]

Maximum bending stress:
\[
\sigma_{\text{max}} = K \frac{M_d}{I}
\]

*\(K\) is the stress concentration factor – normal stress

**Transversal shear**

Nominal shear stress:
\[
\tau_{\text{nom}}
\]

Maximum shear stress:
\[
\tau_{\text{max}} = K_s \tau_{\text{nom}}
\]

*\(K_s\) is the stress concentration factor – transversal shear stress
Stress concentrations: **normal stresses**: bending
Stress concentrations: normal stresses: bending
Reading assignment

• Chapter 6 of textbook
• Review notes and text: ES2001, ES2501
Homework assignment

• As indicated on webpage of our course