STRESS ANALYSIS
ES-2502, C'2012

Lecture 19:
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General information

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Statically indeterminate torque-loaded members

\[ \sum M_x = 0 \]

\[ T - T_A - T_B = 0 \]

Statically indeterminate situation !!
Statically indeterminate torque-loaded members

Compatibility equation: \( \frac{T_A L_{AC}}{J G} = \frac{T_B L_{BC}}{J G} \) (Same angle of twist)
Statically indeterminate torque-loaded members

Solution with: two equations for two unknowns \((T_A \text{ and } T_B)\):

\[
\begin{align*}
(1) \quad & T - T_A - T_B = 0 \\
(2) \quad & \frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0
\end{align*}
\]
Statically indeterminate torque-loaded members: example A

The A-36 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.

**Approach:**

1) Apply equilibrium equations
2) Apply compatibility equations (*two possible methods*)
3) Solve for stresses
Statically indeterminate torque-loaded members: example B

The steel shaft is made from two segments: $AC$ has a diameter of 0.5 in, and $CB$ has a diameter of 1 in. If it is fixed at its ends $A$ and $B$ and subjected to a torque of 500 lb\cdot in, determine the maximum shear stress in the shaft. $G_{st} = 10.8$ Mpsi.
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Statically indeterminate torque-loaded members: example C

The shaft is made from a solid steel section $AB$ and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at $A$, and a torque of $T = 50$ lb$\cdot$ft is applied to it at $C$, determine the angle of twist that occurs at $C$ and compute the maximum shear stress and maximum shear strain in the brass and steel. $G_{st} = 11.5$ Mpsi, $G_{br} = 5.6$ Mpsi.
Statically indeterminate torque-loaded members: example D

The A-36 steel shaft is made from two segments: $AC$ has a diameter of 0.5 in and $CB$ has a diameter of 1 in. If the shaft is fixed at its ends $A$ and $B$ and subjected to a uniform distributed torque of $60 \text{ lb} \cdot \text{in/in}$ along segment $CB$, determine the absolute maximum shear stress in the shaft.

Approach:
1) Apply equilibrium equations
2) Apply compatibility equations
3) Solve for stresses
Statically indeterminate torque-loaded members: example E

If the shaft is subjected to a uniform distributed torque of 20 kN·m/m, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at A and C.

Approach:
1) Apply equilibrium equations
2) Apply compatibility equations
3) Solve for max. stress
Torque-loaded members: stress concentration

Locations producing stress concentrations

Failed component

Keys and keyways
Torque-loaded members: stress concentration

Welded components

Locations with stress concentrations

Locations with stress concentrations

Changes in cross section
Torque-loaded members: stress concentration

\[
\left( \tau_{\text{max}} \right)_{\text{amplified}} = K \tau_{\text{max}} = K \frac{T c}{J}
\]

Stress concentration factor: \( K \)
(amplification factor)

Changes in cross section
Torque-loaded members: stress concentration
Stress concentrations, torsion: example A

The assembly is subjected to a torque of $710 \text{ lb} \cdot \text{in}$. If the allowable shear stress for the material is $\tau_{\text{allow}} = 12 \text{ ksi}$, determine the radius of the smallest size fillet that can be used to transmit the torque.
Stress concentrations, torsion: example A
Reading assignment

• Chapter 5 of textbook
• Review notes and text: ES2001, ES2501
Homework assignment

• As indicated on webpage of our course