STRESS ANALYSIS
ES-2502, C'2012

Lecture 17:
10 February 2012
General information

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Torsion: shear strains

Shear strains vary linearly within a section:

$$\gamma = \gamma(\rho) = \rho \frac{\gamma_{\text{max}}}{c}$$

The angle of twist $\phi(x)$ increases as $x$ increases.
According to Hook’s law (linear elasticity):

\( \tau = G \cdot \gamma \)

Shear stresses also vary linearly within a section:

\[
\tau = \tau(\rho) = \rho \frac{\tau_{\text{max}}}{c}
\]

Differential Force:

\[ dF = \tau \cdot dA \]

Differential Torque:

\[ dT = \rho (\tau \cdot dA) \]
Torsion formula

Integrating torque:

\[ T = \int_{A} \rho (\tau \cdot dA) = \int_{A} \rho \left( \rho \frac{\tau_{\text{max}}}{c} \right) dA \]

\[ = \frac{\tau_{\text{max}}}{c} \int_{A} \rho^2 dA \]

Define: \[ J = \int_{A} \rho^2 dA \quad \text{Polar area moment of inertia} \]

Torsion formula for stresses: (linear elastic)

\[ \tau_{\text{max}} = \frac{T}{J} c \quad \text{and} \quad \tau = \tau(\rho) = \frac{T \rho}{J} \]
Torsion formula: solid circular bar
Linear variation of shear stress

Shear stress varies linearly along each radial line of the cross section.
Torsion formula: polar area moment of inertia

Solid bar

\[ J = \int_A \rho^2 \, dA \]

\[ = \int_0^c \rho^2 (2\pi \rho \, d\rho) \]

\[ = 2\pi \int_0^c \rho^3 \, d\rho = 2\pi \left( \frac{\rho^4}{4} \right)_0^c \]

Solid, circular, section:

\[ J = \frac{\pi}{2} c^4 \]
Torsion formula: polar area moment of inertia

Tubular bar

\[ J = \int_A \rho^2 \, dA \]

\[ = \int_{c_i}^{c_o} \rho^2 \left( 2\pi \, \rho \, d\rho \right) \]

\[ = 2\pi \int_{c_i}^{c_o} \rho^3 \, d\rho = 2\pi \left( \frac{\rho^4}{4} \right)_{c_i}^{c_o} \]

Tubular section:

\[ J = \frac{\pi}{2} \left( c_o^4 - c_i^4 \right) \]
Torsion formula: tubular bar
Linear variation of shear stress

Shear stress varies linearly along each radial line of the cross section.
Torsion: example A

A shaft is made of a steel alloy having an allowable shear stress of $\tau_{\text{allow}} = 12$ ksi. If the diameter of the shaft is 1.5 in., determine the maximum torque $T$ that can be transmitted.

What would be the maximum torque $T'$ if a 1-in. diameter hole is bored through the shaft? Sketch the shear-stress distribution *along a radial line* in each case.
Torsion: example B

The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume (stress) elements located at these points.
Torsion: example C

The solid 30 mm diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.
Torsion: example D

The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter $d$. 
Power transmission

\[ P = T \omega \]

with:

\[ \omega = 2\pi \cdot f \]

\( f \) [Hz]

\( \omega \) [rad/sec]

SI:

\[ 1W = 1N \cdot \frac{m}{s} \]

FPS:

\[ 1hp = 550 \text{ ft} \cdot \frac{lb}{s} \]
The 25 mm diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75$ MPa. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.
Torsion: angle of twist

The angle of twist $\phi(x)$ increases as $x$ increases.
Torsion: angle of twist $\phi$

$BD = \rho \cdot \Delta \phi$

$BD = \gamma \cdot \Delta x$

Shear strain: $\gamma = \rho \frac{d\phi}{dx}$

Therefore: $d\phi = \frac{\gamma}{\rho} dx$
Torsion: angle of twist $\phi$

From before:  

$$d\phi = \frac{\gamma}{\rho} \, dx$$

By Hook’s law:  

$$\gamma = \frac{\tau}{G} = \frac{1}{G} \frac{T}{J} \rho$$

$$\gamma(x, \rho) = \frac{1}{G} \frac{T(x)}{J(x)} \rho$$

Angle of twist:  

$$\phi(x) = \int_{0}^{L} \frac{1}{G} \frac{T(x)}{J(x)} \, dx$$
Torsion: angle of twist $\phi$

Constant torque and cross sectional area

Angle of twist: $\phi(x = L) = \frac{1}{G} \frac{T}{J} \frac{L}{J}$
Torsion: angle of twist $\phi$

Multiple torques

$$\phi = \sum_i \left( \frac{1}{G} \frac{T}{J} \right)_i$$
Torsion: example F

The splined ends and gears attached to the A-36vsteel shaft are subjected to the torques shown. Determine the angle of twist of end \( B \) with respect to end \( A \). The shaft has a diameter of 40 mm.
Reading assignment

• Chapter 5 of textbook
• Review notes and text: ES2001, ES2501
Homework assignment

• As indicated on webpage of our course