We will get started soon...

10 April 2020
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Lecture 10:
Unit 6: tension/compression of slender longitudinal bars: general

10 April 2020
General information

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Strain: example A

A concrete cylinder having a diameter of 6.0 in and a gauge length of 12 in is tested in compression. The results of the test are reported in the table as load versus contraction. Draw stress-strain diagram and estimate modulus of elasticity.

<table>
<thead>
<tr>
<th>Displacement, in</th>
<th>Load, kip</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0006</td>
<td>5.0</td>
</tr>
<tr>
<td>0.0012</td>
<td>9.5</td>
</tr>
<tr>
<td>0.0020</td>
<td>16.5</td>
</tr>
<tr>
<td>0.0026</td>
<td>20.5</td>
</tr>
<tr>
<td>0.0036</td>
<td>25.5</td>
</tr>
<tr>
<td>0.0040</td>
<td>30.0</td>
</tr>
<tr>
<td>0.0045</td>
<td>34.5</td>
</tr>
<tr>
<td>0.0050</td>
<td>38.5</td>
</tr>
<tr>
<td>0.0062</td>
<td>46.5</td>
</tr>
<tr>
<td>0.0070</td>
<td>50.0</td>
</tr>
<tr>
<td>0.0075</td>
<td>53.0</td>
</tr>
</tbody>
</table>
Strain: example A

Compute stress and strain table:

<table>
<thead>
<tr>
<th>Displacement, in</th>
<th>Load, kip</th>
<th>Strain, in/in</th>
<th>Stress, kpsi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0</td>
<td>0.0000000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.0006</td>
<td>5.0</td>
<td>0.000050</td>
<td>0.177</td>
</tr>
<tr>
<td>0.0012</td>
<td>9.5</td>
<td>0.000100</td>
<td>0.336</td>
</tr>
<tr>
<td>0.0020</td>
<td>16.5</td>
<td>0.000167</td>
<td>0.584</td>
</tr>
<tr>
<td>0.0026</td>
<td>20.5</td>
<td>0.000217</td>
<td>0.725</td>
</tr>
<tr>
<td>0.0036</td>
<td>25.5</td>
<td>0.000300</td>
<td>0.902</td>
</tr>
<tr>
<td>0.0040</td>
<td>30.0</td>
<td>0.000333</td>
<td>1.061</td>
</tr>
<tr>
<td>0.0045</td>
<td>34.5</td>
<td>0.000375</td>
<td>1.220</td>
</tr>
<tr>
<td>0.0050</td>
<td>38.5</td>
<td>0.000417</td>
<td>1.362</td>
</tr>
<tr>
<td>0.0062</td>
<td>46.5</td>
<td>0.000517</td>
<td>1.645</td>
</tr>
<tr>
<td>0.0070</td>
<td>50.0</td>
<td>0.000583</td>
<td>1.768</td>
</tr>
<tr>
<td>0.0075</td>
<td>53.0</td>
<td>0.000625</td>
<td>1.874</td>
</tr>
</tbody>
</table>
Strain: example A

Plot data and estimate yield point:

\[ E = \frac{\sigma_{pl} - 0}{\varepsilon_{pl} - 0} \approx 3.27 \times 10^3 \text{ ksi} \]

\[ y = 3177.8x + 0.0159 \]
\[ R^2 = 0.9958 \]

\[ y = -716096x^2 + 3474.9x \]
\[ R^2 = 0.9975 \]
Strain: example B

The $\sigma$–$\varepsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at $A$ has a length of 6.5 in and an approximate cross-sectional area of 0.23 in$^2$ determine its elongation if the foot supports a load of 125 lb, which causes a tension in the tendon of 343.7 lb.
Poisson’s ratio:

\[ \nu = - \frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{longitudinal}}} \]
Shear stress ↔ strain

Pure shear

Hook’s law for shear: \( \tau = G \gamma \)

with \( G = \frac{E}{2(1 + \nu)} \) (shear modulus)

\( \gamma \)

\( \text{Stress} \)

\( \tau_{xy} \)

\( \gamma_{xy} \)

\( \frac{\pi}{2} - \gamma_{xy} \)

\( x \)

\( y \)

\( \frac{\gamma_{xy}}{2} \)

\( \frac{\gamma_{xy}}{2} \)

\( x \)

\( y \)
Strain: example C

A bar made of ASTM A-36 steel has the dimensions shown. If the axial force of \( P = 80 \text{ kN} \) is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.
Axial load

Note distortion lines: follow Saint-Venant’s principle

Figure: 04-01-UN-A
Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant’s principle.

Figure: 04-01-UN-B
Notice how the lines on this rubber membrane distort after it is stretched. The localized distortions at the grips smooth out as stated by Saint-Venant’s principle.
Axial load: Saint-Venant’s principle

Internal distribution of stresses at various sections

Section \(a-a\)

Section \(b-b\)

Section \(c-c\)

\[\sigma_{\text{avg}} = \frac{P}{A}\]
Axial load: Saint-Venant’s principle

Section $c-c$

Nominal stress

Uniform stress distribution

\[ \sigma_{\text{avg}} = \frac{P}{A} \]
Axial load: Saint-Venant’s principle

In your analyses, select locations (sections / points) located away from regions that are subjected to load application (to eliminate “end” effects).

**Saint-Venant’s principle:** stresses and strains within a section will approach their nominal values as the section locates away from regions of load application.

\[ \sigma_{avg} = \frac{P}{A} \]

- Section \( c-c \)
- Nominal stress
Elastic deformation of an axially loaded member

Section is a function of position

\[ \sigma = \frac{P(x)}{A(x)} \quad \text{and} \quad \varepsilon = \frac{d\delta}{dx} \]

Therefore, \[ d\delta = \frac{P(x) \, dx}{A(x) \, E} \]

\[ \delta = \int_{0}^{L} \frac{P(x)}{A(x) \, E} \, dx \]
Elastic deformation of an axially loaded member

Constant load and cross-sectional area

Elastic deformation:

\[ \delta = \int_0^L \frac{P(x)}{A(x)E} \, dx = \frac{P}{AE} \int_0^L dx = \frac{PL}{AE} \]
Elastic deformation of an axially loaded member

Bar subjected to multiple axial loads

Elastic deformation:

\[ \delta = \sum_i \left( \frac{P L}{A E} \right)_i \]
Elastic deformation of an axially loaded member

Procedure for analysis

Internal loads

$P_{AB} = 5\ \text{kN}$

$P_{BC} = 3\ \text{kN}$

$P_{CD} = 7\ \text{kN}$
Elastic deformation of an axially loaded member
Procedure for analysis

Sign convention:
+ tension and elongation
− compression and contraction

Internal loads
Axial load: example D

The assembly shown consists of an aluminum tube $AB$ having a cross sectional area of $400 \, \text{mm}^2$. A steel rod having a diameter of $10 \, \text{mm}$ is attached to a rigid collar and passes through the tube. If a tensile load of $80 \, \text{kN}$ is applied to the rod, determine the displacement of the end $C$ of the rod. Elastic modules: $E_{\text{steel}} = 200 \, \text{GPa}$ and $E_{\text{alum}} = 200 \, \text{GPa}$

Approach:

1) Determine internal loading
2) Compute displacement
Axial load: example D

Displacement of $C$:

$$\delta_C = \delta_B + \delta_{C/B}$$
Axial load: example D

(1) Internal loading

\[ P_{AB} = 80 \text{ kN} \]

\[ P_{BC} = 80 \text{ kN} \]

(2) \( \rightarrow \) find displacement at \( C \)

\[ 400 \text{ mm} \]

\[ 600 \text{ mm} \]
Reading assignment

• Chapters 3 and 4 of textbook
• Review notes and text: ES2001, ES2501
Homework assignment

- As indicated on webpage of our course