Lecture 07: Stress and Strain

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General information

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Strain

Observe what happened to the white line segments in this tensile test experiment.

Figure: 02-01-A-UN
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Figure: 02-01-B-UN
Note the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.
**Strain**: definition: change in length per unit length

**Normal** strain
**Strain**: definition: change in length per unit length

**Normal strain**

Average normal strain:

\[ \varepsilon_{\text{avg}} = \frac{\Delta s' - \Delta s}{\Delta s} \]

Normal strain:

\[ \varepsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s} \]
**Strain**: definition: change in length per unit length

**Shear strain**

![Diagram of undeformed and deformed body](image)
Strain: definition: change in length per unit length

Shear strain

Shear strain:

\[ \gamma_{nt} = \frac{\pi}{2} - \lim_{B \rightarrow A \text{ along } n} \lim_{C \rightarrow A \text{ along } t} \theta' \]
Cartesian strain components

Before deformations

Differential element or cube (undeformed)
Dimensions: $\Delta x, \Delta y, \Delta z$
Cartesian strain components

Differential element or cube (deformed)

New dimensions: \( \Delta x + \varepsilon_x (\Delta x) \)
\( \Delta y + \varepsilon_y (\Delta y) \)
\( \Delta z + \varepsilon_z (\Delta z) \)

After deformations
Cartesian strain components

Differential element or cube (deformed)

Original dimensions

Elongations

New dimensions:
\[ \Delta x + \varepsilon_x (\Delta x) = (1 + \varepsilon_x ) \Delta x \]
\[ \Delta y + \varepsilon_y (\Delta y) = (1 + \varepsilon_y ) \Delta y \]
\[ \Delta z + \varepsilon_z (\Delta z) = (1 + \varepsilon_z ) \Delta z \]

Approximate angles between sides:
\[ \frac{\pi}{2} - \gamma_{xy}, \quad \frac{\pi}{2} - \gamma_{yz}, \quad \frac{\pi}{2} - \gamma_{xz} \]
Cartesian strain components

1) Normal strains cause a change in volume of the element
2) Shear strains cause a change in its shape
3) Normal and shear strains occur simultaneously during deformation
4) State of strain at a point on a body requires specifying 6 strain components: $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$, and $\gamma_{xy}$, $\gamma_{yz}$, $\gamma_{xz}$
Typical strain distributions generated inside a bolted assembly

Polarized light used in experiment shown: bolted assembly

Strains are related to stresses in the materials

Strains can be measured and stresses estimated from strains
Typical strain distributions generated inside a bolted assembly

Polarized light used in experiment shown: component in compression

Strains are related to stresses in the materials

Strains can be measured and stresses estimated from strains
Strain: example A

When force $\mathbf{P}$ is applied to the rigid lever arm $ABC$ shown, the arm rotates counterclockwise about pin $A$ through an angle of $0.05^\circ$. Determine the normal strain developed in wire $BD$.

**Approach:**

1) Define geometry
2) Determine change in geometry
3) Compute required strains
Strain: example A

When force $P$ is applied to the rigid lever arm $ABC$ shown, the arm rotates counterclockwise about pin $A$ through an angle of $0.05^\circ$. Determine the normal strain developed in wire $BD$. 

Change in geometry & Computation of normal strain
An air-filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball’s diameter becomes 7 in, determine the average normal strain in the rubber

**Approach:**

1) Define geometry
2) Determine change in geometry
3) Compute required strains
Strain: example B

An air-filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball’s diameter becomes 7 in, determine the average normal strain in the rubber.

Change in geometry & Computation of normal strain

\[ d_o = 6 \text{ in} \]

\[ d_o + \Delta d \]

\[ \text{change in length of circumference} \rightarrow \varepsilon \]
Strain: example C

If the unstretched length of the bowstring is 35.5 in., determine the average normal strain in the string when it is stretched to the position shown.
Strain: example D

Part of a control linkage for an airplane consists of a rigid member $CBD$ and a flexible cable $AB$. If a force is applied to the end $D$ of the member and causes it to rotate by $\theta = 0.3^\circ$, determine the normal strain in the cable. Originally the cable is unstretched.
Part of a control linkage for an airplane consists of a rigid member $CBD$ and a flexible cable $AB$. If a force is applied to the end $D$ of the member and causes a normal strain of 0.0035 mm/mm, determine the displacement of point $D$. Originally the cable is unstretched.
Reading assignment

• Chapter 1 of textbook
• Review notes and text: ES2001, ES2501
Homework assignment

• As indicated on webpage of our course