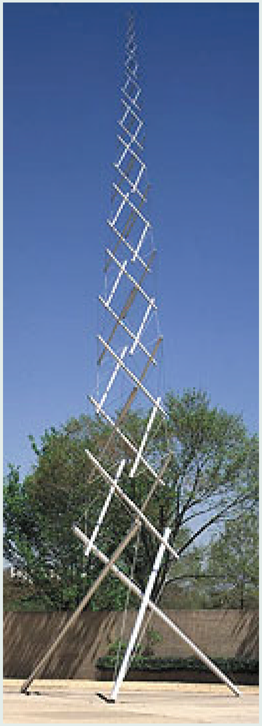
A Literature Review of Tensegrity

Brigitte Servatius



1. 41 Papers on Tensegrity

1 Tensegrity Frameworks, Roth, Whiteley, 1981
2 On generic rigidity in the plane., Lovsz, Yemini, 1982
3 Survey of research work on structures., Minke, 1982
4 Cones, infinity and 1-story buildings, Whiteley, 1983
5 Statics of frameworks, Whiteley, 1984
6 Prismic tensigrids, Hinrichs, 1984
7 Scene analysis and motions of frameworks, Whiteley, 1984
8 1-story buildings as tensegrity frameworks, Recski, 1986
9 A Fuller explanation. Edmondson, 1987.
10 Immobile kinematic chains, Kuznetsov, 1989

11 Rigidity and polarity. II, Whiteley, 1989
12 Duality between plane trusses and grillages, Tarnai, 1989
13 1st infinitesimal mechanisms, Calladine, Pellegrino, 1991
14 1-story buildings as tensegrity frameworks II, Recski 1991
15 1-story buildings as tensegrity frameworks III Recski 1992
16 Combinatorics to statics, 2nd survey, Recski, 1992
17 Rigidity, Connellly, 1993
18 Globally rigid symmetric tensegrities,
Connelly, Terrell, 1995
19 Graphs, digraphs, and the rigidity of grids, Servatius, 1995
20 Second-order rigidity and prestress stability,
Connelly, Whiteley 1996

21 Controllable tensegrity, Skelton and Sultan 1997
22 Tensegrities and rotating rings of tetrahedra, Guest, 2000
23 Dynamics of the shell class of tensegrity structures, Skelton 2001
24 Cyclic frustum tensegrity modules, Nishimura, Murakami, 2001
25 Modelling and control of class NSP tensegrity structures, Kanchanasaratool, Williamson, 2002
26 Two-distance preserving functions, Khamsemanan, Connelly, 2002
27 Tensegrity and the viscoelasticity of the cytoskeleton, Caladas, et.al. 2002
28 Motion control of a tensegrity platform, Kanchanasaratool, Williamson, 2002
29 Stability conjecture in the theory of tensegrity structures, Volokh, 2003
30 Equilibrium conditions of a tensegrity structure, Williamson, Skelton, Han, 2003

31 Tensegrities, Heunen, van Leijenhorst, Dick, 2004

32 Algebraic tensegrity form-finding, Masic, Skelton, Gill, 2005

33 Towards understanding tensegrity in cells, Shen, Wolynes, 2005

34 Structures in hyperbolic space, Connelly, 2006

35 dynamic analysis of a planar 2-DOF tensegrity mechanism, Arsenault, Gosselin, 2006

36 Modeling virus self-assembly pathways, Sitharam, Agbandje-Mckenna, 2006

37 Improving the DISPGB algorithm using the discriminant ideal, Manubens, Montes, 2006

38 From graphs to tensegrity structures: geometric and symbolic approaches, de Guzman, Orden, 2006

39 Stability conditions for tensegrity structures, Zhang, Ohsaki, 2007

41 Papers on Tensegrity

Highlights

Some Questions

Home Page

Title Page

Page 6 of 79

Go Back

Full Screen

Close

Quit

Tensegrity frameworks

Roth, B.; Whiteley, W.
Trans. Amer. Math. Soc. 265 (1981), no. 2, 419–446.

A tensegrity framework $G(p)$ is an abstract graph G with each edge called a bar, cable, or strut, together with an assignment of a point p_i in Euclidean space R^n for each vertex of G. Let $p = (\cdots, p_i, \cdots) \in R^{nv}$, where v is the (finite) number of vertices of G. A flexing of $G(p)$ is a continuous (or equivalently, analytic) path of the vertices $x(t)$, $0 \leq t \leq 1$, such that $x(0) = p$, bars stay the same length, cables do not increase in length, and struts do not decrease in length.

If all flexings of $G(p)$ are restrictions of rigid motions of R^n, then $G(p)$ is said to be rigid, or not a mechanism. For a tensegrity framework an infinitesimal motion of $G(p)$ is an assignment of vectors μ_i to each vertex of G such that, for each bar, cable, or strut, $(p_i - p_j)(\mu_i - \mu_j)$ is $= 0$, ≤ 0, or ≥ 0, respectively. If $\mu = (\cdots, \mu_i, \cdots)$ is the restriction of the derivative of a rigid motion of R^n, is said to be trivial. If $G(p)$ has only trivial infinitesimal motions then $G(p)$ is said to be infinitesimally rigid.

A stress for $G(p)$ is an assignment of a scalar $\omega_{\{i,j\}}$ for each edge of G such that for each vertex i, $\sum_j \omega_{\{i,j\}}(p_i - p_j) = 0$, where the sum is over the edges adjacent to i. A stress is also required to be nonpositive on cables and nonnegative on struts. The stress is said to be proper if the stresses on all the cables and struts are nonzero.

The first four sections of the paper are basically a review of notation and a description of equivalent notions of rigidity and infinitesimal rigidity in terms of tensegrity structures. The basic results here are essentially a review in these terms of the papers by H. Gluck

The basic result of the paper under review is Theorem 5.2, which says that a tensegrity framework $G(p)$ is infinitesimally rigid if and only if $G(p)$ has a proper stress and $\overline{G}(p)$ is infinitesimally rigid, where $\overline{G}$ is the graph obtained by changing all the cables, struts, and bars to bars. This result is used to prove analogues for tensegrity frameworks of Gluck's result about bar frameworks. Here, for instance, it can only be said that the set $\{p \in R^{nv} \colon G(p)$ is infinitesimally rigid$\}$is open in R^{nv} and not that it is dense (Theorem 5.4). Another important result is Theorem 5.10, which says roughly that if $G(p)$ is infinitesimally rigid, if each p_i is replaced by $Lp_i = q_i$, where L is a projective map of R^n and if certain cables and struts are changed to get G', then the new $G'(q)$ is infinitesimally rigid.

Section 6 specializes to tensegrity frameworks in the plane. Applying their stress criterion for infinitesimal rigidity, the authors provide another proof of the infinitesimal rigidity of Cauchy polygons described in the reviewer's paper, as well as many other related frameworks.

Section 7 deals with analogous results in R^3, where many of the tensegrity frameworks of the type considered by R. Buckminster Fuller are shown to be infinitesimally rigid.

Back to Main List

On generic rigidity in the plane.

Lovsz, L.; Yemini, Y.

SIAM J. Algebraic Discrete Methods 3 (1982), no. 1, 91–98.

A finite graph whose vertices are points in Euclidean space is viewed as a collection of rigid bars and universal joints (a "tensegrity structure", a la Buckminster Fuller). The problem of assigning a "degree of floppiness" to a graph in the plane in such a way that degree 0 means "rigid" is dealt with (a triangle is rigid, a square is not); and the authors use matroid theory to prove G. Laman's theorem showing how to compute this number.

The matroid methods are also used to show that 6-connected graphs are rigid in the plane but 5-connected graphs need not be. The techniques are more algebraic than those used by other authors in dealing with similar problems, and hope is expressed in the present paper that higher-dimensional analogues to Laman's theorem will now be easier to find. Finally the authors conjecture that any $n(n+1)$-connected graph will be rigid in n-dimensional Euclidean space.

Back to Main List

Survey of research work on structures

Minke, Gernot
Structural Topology 1982, no. 6, 21–32.

This report concerns the work of the Research Laboratory for Experimental Building at the University of Kassel, in West Germany. We describe the design and use of equipment for soap film simulation of minimal length-sum networks connecting configurations of given points in the plane.
This research has application to city planning, in the design of roadway and pedestrian networks. Subsequent sections deal with the research on tensegrity structures, with space grid cable structures, with minimum-cost fabric-covered frame structures, and with grid shell and pneumatic structures.

Back to Main List

Cones, infinity and 1-story buildings.

Whiteley, Walter
Structural Topology 1983, no. 8, 53–70.

Consider a 1-story building constructed with a series of vertical columns (bars and joints possibly of different lengths), a roof framework connecting the tops of the columns, plus a minimum of three additional wall braces going to the tops of some columns. We examine the static rigidity (or equivalently, infinitesimal rigidity) of this framework by viewing it as a 'cone' from the roof to the point at infinity at the common end of all the vertical columns. We conclude that the framework is statically rigid if and only if the orthogonal projection of the roof onto a horizontal plane is statically rigid in the plane, and the three wall braces do not meet a common vertical line.

This analysis is extended to tensegrity structures (with cables), to structures with extra wall braces (and fewer roof braces) and to multi-story buildings with vertical columns. In all cases the static rigidity of the structure is tested by the static rigidity of the appropriate single plane projection. We conclude with some mathematical recreation. We introduce the spherical model for statics as a truly projective geometry study.

Back to Main List

Infinitesimally rigid polyhedra. I. Statics of frameworks.

Whiteley, Walter
Trans. Amer. Math. Soc. 285 (1984), no. 2, 431–465.
The reviewer can hardly do better than use the author's abstract to provide a sample of the good things in store for the reader. "In this paper the concept of static rigidity for frameworks is used to describe the behavior of bar and joint frameworks built around convex (and other) polyhedra." For example, the author proves that a triangulated polyhedron can be rigid even when it has vertices interior to its natural faces, extending Aleksandrov's extension of Cauchy's rigidity theorem. He also studies the "static rigidity of tensegrity frameworks (with cables and struts in place of bars) (and) detailed analogues of Aleksandrov's theorem for convex 4-polytopes".

All this and more is presented with attention paid to the shape and history of this growing part of mathematics and with a care which brings accuracy to an area often plagued by theorems which are only almost true.

Back to Main List

Prismic tensigrids.

Hinrichs, Lowell A.
Structural Topology 1984, no. 9, 3–14.

In this paper the author studies symmetric rigid structures in space built from struts and cables and so arranged that just one strut meets each vertex. It is this last property which makes these objects interesting sculptures—the viewer wonders why they don't collapse.

The regularity imposed is that the group G of symmetries act transitively on the vertices of the structure. In consequence, G acts transitively on the struts. The author uses a theorem on the rigidity of general tensegrities due to B. Roth and W. Whiteley to find all structures in which the cables are partitioned into two G-orbits.

As is traditional for this journal, the illustrations are clear, and invite the reader to build models.

Back to Main List

A correspondence between scene analysis and motions of frameworks.

Whiteley, Walter
Discrete Appl. Math. 9 (1984), no. 3, 269–295.

It is known that projections of a spherical polyhedron into a plane correspond to a static stress on the projection. Here a different correspondence is established between a plane picture of a “scene” and a plate and bar framework in the plane. This provides an equivalence between these two areas of study. This also allows a correspondence between scenes with occlusion and tensegrity frameworks where the linear systems involve inequalities.

Back to Main List

One-story buildings as tensegrity frameworks.

Chakravarty, N.; Holman, G.; McGuinness, S.; Recski, A.
Structural Topology No. 12 (1986), 11–18.

In this short, accessible paper the authors re-prove theorems of the reviewer and H. Crapo on the bracing of plane grids of squares and of one-story buildings.

They then generalize to allow cables and struts as bracing elements. Thus, for example, a plane grid of squares is rigid if and only if the directed bipartite graph describing the placement of the struts and cables is strongly connected. This result may also be found in a book by J. A. Baglivo and J. E. Graver.

The proofs rely on the direct analysis of some simple systems of equations rather than on the machinery (or language) of matroid theory.

Back to Main List

A Fuller explanation.

The synergetic geometry of R. Buckminster Fuller
Edmondson, Amy C.
Design Science Collection. A Pro Scientia Viva Title.
Birkhauser Boston, Inc., Boston, MA, 1987.

This book is a disciple's affectionate tribute to R. Buckminster Fuller (1895–1983), "inventor, architect, engineer, and philosopher".
It begins with a portrait and a view of his most famous building: the great geodesic dome in Montreal which was built in 1967 as the U.S. Pavilion. He was deeply interested in Platonic and Archimedean polyhedra, especially the cuboctahedron, which he quaintly renamed "vector equilibrium". The figures on page 115 illustrate the cuboctahedral numbers $1 + \sum_{f=1}^{n}(10f^2 + 2)$ as clusters of congruent balls

Chapter 15 describes Fuller's concept of "tensegrity", the most remarkable instance of which is a skeletal object consisting of 30 rigid struts firmly held together by 90 threads. It is symmetrical by the icosahedral group of rotations. The 60 ends of the 30 struts are the vertices of one of the two chiral Archimedean solids, the snub dodecahedron. The threads run along 90 of the 150 edges, namely the 60 sides of the 12 pentagonal faces and 30 shared by pairs of triangles. Nearly parallel to those 30, and nearly three times as long, are 30 diagonals which are likewise reversed by the 30 half-turns belonging to the icosahedral group. These 30 diagonals are the positions for the 30 struts. The same chapter provides a fuller explanation for the structure of "geodesic domes", based on tessellations of a sphere by $20T$ nearly congruent, nearly equilateral triangles, where $T = b^2 + bc + c^2$, b and c being nonnegative integers

Page 265 shows the "Dymaxion map", in which the whole world is projected onto the faces of a regular icosahedron which is then cut along certain edges and spread out flat. He conceived this idea in 1943, simultaneously with Professor Irving Fisher of Yale. Fisher placed the North and South poles at two opposite vertices, with the unhappy result that some countries (such as Algeria) are "ripped apart" by an edge separating two faces. "It took Fuller two years of experimenting to find an orientation in which all twelve icosahedral vertices land in the ocean—an essential requirement if land masses are not to be ripped apart."
Reviewed by H. S. M. Coxeter

Back to Main List

On immobile kinematic chains and a fallacious matrix analysis.

Kuznetsov, E. N.
Trans. ASME J. Appl. Mech. 56 (1989), no. 1, 222–224.

T. Tarnai asked the following questions about pin jointed bar frameworks: "(1) What criterion determines whether self-stress stiffens an assembly which is both statically and kinematically indeterminate? and (2) How can matrix methods be used to decide whether kinematical indeterminacy takes the form of an infinitesimal or a finite mechanism?"
S. Pellegrino and C. R. Calladine attempted to answer these questions.

In the present paper the author points out, among other things, that the analysis by Pellegrino and Calladine is at the least inadequate because it fails to provide information about whether a certain quadratic form is positive definite, in order to detect when a certain potential function (associated to the stress-strain characteristics on the members) is at a local minimum. This is one very natural test to detect the stability of a structure in a special position.

In the reviewer's opinion, part of the confusion comes from not making the distinction between "pre-stress stability", where the second derivative test works for the potential function involved, and "second-order rigidity", where second-order terms are introduced and are compatible with the geometric distance constraints. If a structure is pre-stress stable, then it is second-order rigid, but not conversely. See a forthcoming paper by the reviewer and W. Whiteley.

Back to Main List

Rigidity and polarity. II. Weaving lines and tensegrity frameworks.

Whiteley, Walter

Geom. Dedicata 30 (1989), no. 3, 255–279.

A weaving is a finite collection of straight lines in the plane, where certain pairs of the lines are designated so that one passes "over" or "under" the other at the crossing. A lifting of a weaving is a corresponding collection of straight lines in 3-space that projects orthogonally onto the given weaving lines such that the crossings have the corresponding weaving patterns. One kind of problem is to determine when a weaving has a lifting.

The author explores the question of what happens when a configuration of weaving lines is polarized in the plane. Lines of the weaving correspond to points, and crossings of the weaving correspond to lines between corresponding points. Surprisingly, the proper structure to consider for the polar configuration is a tensegrity framework, which is a collection of points in the plane with certain pairs of those points designated as either a "cable" or a "strut". A strict infinitesimal flex of a tensegrity framework is the assignment of a vector $\mathbf{p}_i'$ to each point $\mathbf{p}_i$ of the tensegrity framework such that $(\mathbf{p}_i - \mathbf{p}_j)(\mathbf{p}_i' - \mathbf{p}_j') < 0$ for (i, j) a cable, and $(\mathbf{p}_i - \mathbf{p}_j)(\mathbf{p}_i' - \mathbf{p}_j') > 0$ for (i, j) a strut. This is a rather strong form of nonrigidity for the tensegrity framework, where the cables shorten and the struts lengthen. As a consequence of the polarity, the author shows that a weaving has a strict lifting, where none of the lifted lines intersect, if and only if the corresponding polar tensegrity framework has a strict infinitesimal flex.

The author also explores similar problems with "grillages," which are configurations of lines in 3-space, where certain pairs are forced to intersect.

Back to Main List

Duality between plane trusses and grillages.

Tarnai, T.

Internat. J. Solids Structures 25 (1989), no. 12, 1395–1409.

In the paper, projective plane duality, that is, a point-to-line, line-to-point, incidence-to-incidence correspondence between plane trusses and grillages of simple connection is treated. By means of linear algebra it is proved that the rank of the equilibrium matrix of plane trusses and grillages does not change under projective transformations and polarities; consequently the number of infinitesimal inextensional mechanisms and the number of independent states of self-stress are preserved under these transformations. The results obtained are also applied to structures with unilateral constraints, and by using several examples it is shown that plane tensegrity trusses have projective dual counterparts among grillages which can be physically modelled with popsicle sticks by weaving."

Back to Main List

First-order infinitesimal mechanisms.

Calladine, C. R., Pellegrino, S.
Internat. J. Solids Structures 27 (1991), no. 4, 505–515.
This paper discusses the analytical conditions under which a pin-jointed assembly, which has s independent states of self-stress and m independent mechanisms, tightens up when its mechanisms are excited. A matrix algorithm is set up to distinguish between first-order infinitesimal mechanisms (which are associated with second-order changes of bar length) and higher-order infinitesimal or finite mechanisms.

It is shown that, in general, this analysis requires the computation of s quadratic forms in m variables, which can be easily computed from the states of self-stress and mechanisms of the assembly. If any linear combination of these quadratic forms is sign definite, then the mechanisms are first-order infinitesimal. An efficient and general algorithm to investigate these quadratic forms is given. The calculations required are illustrated for some simple examples. Many assemblies of practical relevance admit a single state of self-stress ($s = 1$), and in this case the algorithm proposed is straightforward to implement. This work is relevant to the analysis and design of pre-stressed mechanisms, such as cable systems and tensegrity frameworks.

Back to Main List

One-story buildings as tensegrity frameworks II

Recski, Andras
Structural Topology No. 17 (1991), 43–52.

In a paper with the same title as this one, N. Chakravarti et al. showed that the minimal number of diagonal cables needed to rigidify a $k \times l$ one-story building is $k + l - 1$ (when $k, l \geq 2$ and $k + l \geq 5$). In this paper the author shows which sets of that size do the job in two particular interesting cases.

As usual, regard the cables as edges in the bipartite graph $K(A, B)$, where $|A| = k$ and $|B| = l$. When the subgraph corresponding to $k + l - 1$ bracing cables is not a tree the cables rigidify the building if and only if $k - l = 1$ and the subgraph is a directed circuit with $2\min(k, l)$ vertices.

When the cables are parallel they rigidify the grid if and only if, for every proper subset X of A, $|N(X)| > (k/l)|X|$, where $N(X)$ denotes the set of those vertices in B adjacent to at least one vertex in X. The proofs are straightforward. The paper contains interesting examples.

Back to Main List

One-story buildings as tensegrity frameworks. III.

Recski, Andras; Schwaerzler, Werner
Discrete Appl. Math. 39 (1992), no. 2, 137–146.

The authors of Parts I and II showed that the minimal number of diagonal cables needed to rigidify a $k \times l$ one-story building is $k + l - 1$ (when $k, l \geq 2$ and $k + l \geq 5$), and showed which sets of that size do the job in two particular interesting cases. In this paper the authors characterize the minimum systems in the general case.

As usual, regard the cables as forming a directed subgraph G of the bipartite graph $K(A, B)$, where $|A| = k$ and $|B| = l$. G has an edge from $a \in A$ to $b \in B$ just when there is a cable between the northeast and southwest corners of the corresponding room in the building. The edge goes the other way when the cable joins the other two corners.

Suppose G is not connected. Then G is rigid if and only if it has two asymmetric components each of which is strongly connected. (If the components of G have vertex sets $A_1, A_2 \subset A$ and $B_1, B_2 \subset B$ then these components are asymmetric when $|A_i| \times l \neq |B_i| \times k$, $i = 1, 2$.)

Suppose G is connected. An AB-path is a directed path starting at some $x \in A$ and ending at some $y \in B$. Let $N(X)$ be the set of endpoints of AB-paths starting in $X \subset A$. Similarly, define a BA-path and $N(Y)$. Then G is rigid if and only if $|N(X)| \times k > |X| \times l$ for all proper subsets X of A or $|N(Y)| \times l > |Y| \times k$ for all proper subsets Y of B. The proof uses a generalization of a generalization of the Koenig-Hall theorem on the existence of perfect matchings in a bipartite graph.

This paper should mark the end of the sequence of papers by Recski et al. on this subject.

Back to Main List

Applications of combinatorics to statics—a second survey.

Recski, Andras
Discrete Math. 108 (1992), no. 1-3, 183–188.

Some recent results are presented concerning the algorithmic aspects of 2-dimensional generic rigidity and 1-story buildings as tensegrity frameworks. Most of these results were obtained after the completion of the first survey.

Back to Main List

Rigidity.

Connelly, Robert
Handbook of convex geometry, Vol. A, B, 223–271, North-Holland, Amsterdam, 1993.

This chapter surveys the intersection between rigidity and convex geometry. The central result is Cauchy's rigidity theorem (1813), which says: "Two convex polyhedra comprised of the same number of equal similarly placed faces are superposable or symmetric." There are two main categories for generalizations; one is in the category of polyhedra and similar discrete objects while the other is in the category of appropriate smooth surfaces. The author gives a very nice and up-to-date survey of the former with an extensive list of literature. A nice feature is that a comparison between the two categories of generalizations is also given.

The author begins by discussing Dehn's (1916) infinitesimal version of Cauchy's theorem and Aleksandrov's (1958) extension. Dehn's theorem says: "Given a convex compact polytope p in three-space with all faces triangles, if one forms the associated bar-framework $G(p)$ by taking the edges as bars and vertices as joints, the $G(p)$ is infinitesimally rigid." When not all faces of p are triangles and one triangulates p by adding new vertices only on the edges of p, Aleksandrov proved that the associated bar framework of the triangulation remains infinitesimally rigid in three-space. The author describes in great detail the various generalizations of these two theorems by present-day authors:

In another direction, Dehn's theorem and Aleksandrov's theorem have also been generalized to higher dimensions. This led to the proof of Barnette's lower bound theorem for triangulated convex polytopes and a new lower bound theorem for nontriangulated convex polytopes.

The relatively new concepts of second-order rigidity and pre-stress stability and their connections with convex surfaces are also discussed extensively in the final section. Other areas covered include Gruenbaum and Shephard's conjectures concerning rigidity of certain tensegrity frameworks in the plane as well as Maxwell-Cremona theory and spider webs.

There are many different concepts of rigidity. In the past this has caused considerable confusion. The author gives very clear definitions of various types of rigidity and their relationships.

Back to Main List

Globally rigid symmetric tensegrities

Connelly, R.; Terrell, M.
Structural Topology No. 21 (1995), 59–78.

L. A. Hinrichs defined a class of symmetric tensegrity frameworks $P_n(j,k)$, $n = 3, 4, \cdots$, and $j, k = 1, 2, \cdots, n-1$. He called them prismic tensegrids. The authors remark that these frameworks are never infinitesimally rigid. However, they prove that $P_n(j,k)$ is globally rigid iff $k = 1$ or $k = n-1$. Since global rigidity implies rigidity, this also shows that $P_n(j,1)$ and $P_n(j,n-1)$ are rigid. This is the main result of the paper.
The authors also raise the question of the rigidity of $P_n(j,k)$ for other values j and k. Since the method used in this paper does not apply, the authors also propose some possible lines of attack. The proof of the main theorem uses some general methods developed by Connelly and Connelly and W. Whiteley

Back to Main List

Graphs, digraphs, and the rigidity of grids.

Servatius, Brigitte
UMAP J. 16 (1995), no. 1, 43–69.

Here is pretty play with problems about the bracing of plane grids of squares for undergraduates who know enough linear algebra to be comfortable with the rank of a (simple) system of linear equations.

After an algebraic prologue, graph theory assumes center stage. No prerequisites are needed. The author gently introduces bipartite graphs and shows how their connectedness is equivalent to the rigidity of the braced grids to which they correspond. Trees and cycles occur naturally when the plot turns to minimal and minimally redundant bracings. Cables, tensegrity and directed graphs follow in act three.

This 23-page paper contains 21 useful exercises together with their solutions, notes for the instructor and suggestions for further reading, all without seeming rushed. It's an expository tour de force.

Back to Main List

Second-order rigidity and prestress stability for tensegrity frameworks.

Connelly, Robert; Whiteley, Walter
SIAM J. Discrete Math. 9 (1996), no. 3, 453–491.

A tensegrity framework is one with cables, bars and struts. The authors define two concepts of rigidity for such a framework—prestress stability and second-order rigidity. The concept of prestress stability is borrowed from structural engineering. The authors also exhibit a hierarchy of rigidity—first-order rigidity implies prestress stability implies second-order rigidity implies rigidity for any framework. A series of examples also show that none of these implications can be reversed.

It is well known that first-order rigidity has a duality exhibited in the equivalence of the so-called static rigidity and infinitesimal rigidity. In this paper a duality is also developed for second-order rigidity. Using this duality, the authors develop a second-order stress test which states: A second-order flex exists if and only if for every proper self-stress of the framework the quadratic form it defines is nonpositive when evaluated at the given first-order flex.

A convex b-c polygon is a framework in the plane with points the vertices of a convex polygon, bars on the edges and cables inside connecting certain pairs of vertices. Roth conjectured that a b-c polygon is rigid if and only if it is first-order rigid. Using the second-order stress test, the authors prove the contrapositive form of Roth's conjecture in its full generality. These results are the main contributions of the paper.

Back to Main List

Controllable tensegrity, a new class of smart structures

Skelton and Sultan
Smart structures and materials 1997: mathematics and control in smart structures (San Diego, CA), 166–177, Proc. SPIE, 3039, SPIE, Bellingham, WA, 1997.

Back to Main List

Tensegrities and rotating rings of tetrahedra: a symmetry viewpoint of structural mechanics.

Guest, S. D.

Science into the next millennium: young scientists give their visions of the future, Part II.
R. Soc. Lond. Philos. Trans. Ser. A Math. Phys. Eng. Sci. 358 (2000), no. 1765, 229–243.

Symmetry is a common attribute of both natural and engineering structures. Despite this, the application of symmetry arguments to some of the basic concepts of structural mechanics is still a novelty. This paper shows some of the insights into structural mechanics that can be obtained through careful symmetry arguments, and demonstrates how such arguments can provide a key to understanding the paradoxical behaviour of some symmetric structures.

Back to Main List

Dynamics of the shell class of tensegrity structures.

Skelton, Pinaud, and Mingori,
Dynamics and control of structural and mechanical systems.
J. Franklin Inst. 338 (2001), no. 2-3, 255–320.

Back to Main List

Initial shape-finding and modal analyses of cyclic frustum tensegrity modules.

Nishimura, Murakami,
Comput. Methods Appl. Mech. Engrg. 190 (2001), no. 43-44, 5795–5818.

Initial equilibrium and modal analyses of Kenneth Snelson's cyclic frustum tensegrity modules with an arbitrary number of stages are presented.
There are $m(\geq 3)$ bars at each stage. The Maxwell number of the modules is $6 - 2m$ and is independent of the number of stages in the axial direction.
Calladine's relations reveal that there are $2-5m$ infinitesimal mechanism modes. For multi-stage modules the necessary conditions for axial assembly of one-stage modules with the same internal element-forces are investigated.
One-stage modules with geometrically similar frustum modules satisfy the necessary conditions. For pre-stressed configurations, modal analyses were conducted to investigate the mode shapes of infinitesimal mechanism modes.

Back to Main List

Modelling and control of class NSP tensegrity structures.

Kanchanasaratool, Williamson,
Internat. J. Control 75 (2002), no. 2, 123–139.

The method of constrained particle dynamics is used to develop a dynamic model of order $12N$ for a general class of tensegrity structures consisting of N compression members (i.e. bars) and tensile members (i.e. cables).
This model is then used as the basis for the design of a feedback control system which adjusts the lengths of the bars to regulate the shape of the structure with respect to a given equilibrium shape. A detailed design is provided for a 3-bar structure.

Back to Main List

Two-distance preserving functions.

Khamsemanan, Connelly,
Beitraege Algebra Geom. 43 (2002), no. 2, 557–564.

Let c, s be positive real numbers such that $c/s < \frac{\sqrt{5}-1}{2}$, and let $f\colon \mathbb{E}^n \to \mathbb{E}^m, n \geq 2$, be any function such that for all $p, q \in \mathbb{E}^n$ we have: (1) if $|p-q| = c$, then $|f(p) - f(q)| \leq c$ and (2) if $|p-q| = s$, then $|f(p) - f(q)| \geq s$.
As has been proved by K. Bezdek and R. Connelly the latter conditions imply: for all $p, q \in \mathbb{E}^n$ necessarily $|p-q| = |f(p) - f(q)|$ (i.e., f should be a congruence).
The above result is an improvement of a previous result of F. Rad, D. Andreescu and D. Vălcan, where the same theorem has been proved under the more strict condition $c/s < \frac{1}{\sqrt{3}}$.
In the paper under review a new proof of the Bezdek-Connelly result is given which (to quote the authors) "uses the mathematical software Maple and is independent of Rado's proof".

Back to Main List

A cellular tensegrity model to analyse the structural viscoelasticity of the cytoskeleton.

Caladas, et.al.

J. Theoret. Biol. 218 (2002), no. 2, 155–173.

Back to Main List

Motion control of a tensegrity platform.

Kanchanasaratool, Williamson,
Commun. Inf. Syst. 2 (2002), no. 3, 299–324.

The paper begins by defining the general structure of a tensegrity platform as a particular class II tensegrity structure in which at least half the nodes have only one bar attached.
A passive nonlinear constrained particle dynamic (or mass-spring) model is developed as the basis for designing a system for controlling the position and orientation of the structure along a prescribed path by adjusting the lengths of the bars. A neural network "inversion problem" which seeks the (approximate) constant input signal to give a desired steady state output response is formulated. The performance of two path tracking algorithms (a quasi-static algorithm based on an open loop piecewise constant input and a gain scheduling algorithm based on an interpolation of "locally designed" controllers) are examined.

Back to Main List

Stability conjecture in the theory of tensegrity structures.

Volokh,
Stability conjecture in the theory of tensegrity structures, Volokh, 2003
Int. J. Struct. Stab. Dyn. 3 (2003), no. 1, 1–16.

The general problem of the stability of tensegrity structures comprising struts and cables is formulated. It is conjectured that any tensegrity system with totally tensioned cables is stable independently of its topology, geometry and specific magnitudes of member forces.

Back to Main List

Equilibrium conditions of a tensegrity structure.

Williamson, Skelton, Han,
Internat. J. Solids Structures 40 (2003), no. 23, 6347–6367.

This paper characterizes the necessary and sufficient conditions for tensegrity equilibria. Static models of tensegrity structures are reduced to linear algebra problems, after first characterizing the problem in a vector space where direction cosines are not needed.
This is possible by describing the components of all member vectors. While our approach enlarges (by a factor of 3) the vector space required to describe the problem, the advantage of enlarging the vector space makes the mathematical structure of the problem amenable to linear algebra treatment. Using the linear algebraic techniques, many variables are eliminated from the final existence equations.

Back to Main List

Tensegrities.

Heunen, van Leijenhorst, Dick
Nieuw Arch. Wiskd. (5) 5 (2004), no. 4, 279–283.

Back to Main List

Algebraic tensegrity form-finding.

Masic, Skelton, Gill,
Internat. J. Solids Structures 42 (2005), no. 16-17, 4833–4858.

This paper concerns the form-finding problem for general and symmetric tensegrity structures with shape constraints. A number of different geometries are treated and several fundamental properties of tensegrity structures are identified that simplify the form-finding problem.

The concept of a tensegrity invariance (similarity) transformation is defined and it is shown that tensegrity equilibrium is preserved under affine node position transformations.
This result provides the basis for a new tensegrity form-finding tool.
The generality of the problem formulation makes it suitable for the automated generation of the equations and their derivatives.
State-of-the-art numerical algorithms are applied to solve several example problems.
Examples are given for tensegrity plates, shell-class symmetric tensegrity structures and structures generated by applying similarity transformation.

Back to Main List

Towards understanding tensegrity in cells.

Shen, Wolynes,
Phys. Rev. E (3) 72 (2005), no. 4, 041927, 11 pp. 92C37

The cytoskeleton is not an equilibrium structure. To develop theoretical tools to investigate such nonequilibrium assemblies, we study a statistical physical model of motorized spherical particles.

Though simple, it captures some of the key nonequilibrium features of the cytoskeletal networks. Variational solutions of the many-body master equation for a set of motorized particles accounts for their thermally induced Brownian motion as well as for the motorized kicking of the structural elements.
These approximations yield stability limits for crystalline phases and for frozen amorphous structures. The methods allow one to compute the effects of nonequilibrium behavior and adhesion (effective cross-linking) on the mechanical stability of localized phases as a function of density, adhesion strength, and temperature.

We find that nonequilibrium noise does not necessarily destabilize mechanically organized structures.
The nonequilibrium forces strongly modulate the phase behavior and have a comparable effect as the adhesion due to cross-linking. Modeling transitions such as these allows the mechanical properties of the cytoskeleton to rapidly and adaptively change. The present model provides a statistical mechanical underpinning for a tensegrity picture of the cytoskeleton.

Back to Main List

Structures in hyperbolic space.

Connelly,
Math. Appl. (N. Y.), 581, Springer, New York, 2006.

This is an overview of some of the similarities and differences between structures such as frameworks and cabled tensegrities in the hyperbolic plane and hyperbolic space on the one hand and the Euclidean plane, the sphere and Euclidean space on the other hand.
The emphasis is on the Cauchy rigidity theorem, Cauchy arm lemma, Dehn theorem on infinitesimal rigidity, Pogorelov correspondence, and Leapfrog lemma. Several conjectures are posed.

Back to Main List

Kinematic, static and dynamic analysis of a planar 2-DOF tensegrity mechanism.

Arsenault, Gosselin,
Mech. Mach. Theory 41 (2006), no. 9, 1072–1089.

Tensegrity mechanisms are lightweight and deployable and can be accurately modeled. Consequently, they constitute an interesting alternative to conventional mechanisms for some applications.

In this work, the kinematics, statics and dynamics of a planar two-degree-of-freedom tensegrity mechanism are studied. Solutions to the direct and inverse static problems are first presented. Afterwards, the boundaries of the actuator and Cartesian workspaces of the mechanism are computed. The stiffness of the mechanism is then detailed for different situations. Finally, a dynamic model is derived and a preliminary control scheme is proposed.

Back to Main List

Modeling virus self-assembly pathways: avoiding dynamics using geometric constraint decomposition.

Sitharam, Agbandje-Mckenna,
J. Comput. Biol. 13 (2006), no. 6, 1232–1265 (electronic).

We develop a model for elucidating the assembly pathways by which an icosahedral viral shell forms from 60 identical constituent protein monomers.

This poorly understood process is a remarkable example of macromolecular self-assembly occurring in nature and possesses many features that are desirable while engineering self-assembly at the nanoscale.

The model uses static geometric and tensegrity constraints to represent the driving (weak) forces that cause a viral shell to assemble and hold it together. The goal is to answer focused questions about the structural properties of a successful assembly pathway.

Pathways and their properties are carefully defined and computed using computational algebra and geometry, specifically state-of-the-art concepts in geometric constraint decomposition. The model is analyzable and refinable and avoids expensive dynamics.
We show that it has a provably tractable and accurate computational simulation and that its predictions are roughly consistent with known information about viral shell assembly. Justifications for mathematical and biochemical assumptions are provided, and comparisons are drawn with other virus assembly models.

A method for more conclusive experimental validation involving specific viruses is sketched.

Overall, the paper indicates a strong and direct, mutually beneficial interplay between (a) the concepts underlying macromolecular assembly; and (b) a wide variety of established as well as novel concepts from combinatorial and computational algebra, geometry and algebraic complexity.

Back to Main List

Improving the DISPGB algorithm using the discriminant ideal.

Manubens, Montes,
J. Symbolic Comput. 41 (2006), no. 11, 1245–1263.

Given a parametrised family of polynomial ideals, it is well known that for different specialisations the Grbner bases may look very different.
V. Weispfenning introduced for this problem the notion of a comprehensive Gruebner basis which remains a Gruebner basis under every specialisation, and provided an algorithm for its construction. The DISPGB algorithm is a more efficient alternative proposed by the second author.

In the paper under review the authors present a new improved version of the DISPGB algorithm using some recent ideas of Weispfenning, in particular the discriminant ideal distinguishing the different special cases. After a presentation of the underlying theory, the new algorithm is discussed in detail. Finally, two concrete examples are studied, a system from robotics and a system describing a tensegrity problem, and for some further benchmark problems a table with results is given.

Back to Main List

From graphs to tensegrity structures: geometric and symbolic approaches.

de Guzman, Orden,
Publ. Mat. 50 (2006), no. 2, 279–299.
A tensegrity structure is a geometric configuration of points and straight edges in $\mathbb{R}^d$ (typically $d = 2, 3$) such that the whole structure is in a self-tensional equilibrium. The word tensegrity comes from tension and integrity.

The authors study the problem of determining a tensegrity structure, in particular, the problem of building tensegrity structures with a given underlying abstract graph $G = (V, E)$, in a given $\mathbb{R}^d$, solving the following problems: decide whether G can be the underlying graph of a tensegrity structure in $\mathbb{R}^d$; and if a tensegrity with G is possible, characterize the relative position of its vertices.
To solve these problems the authors look for decompositions of tensegrities into basic instances, called atoms, by decomposing the graph G into the smallest graphs that can underlie a tensegrity.

To do that, they present two different approaches: (1) to look at the geometric structure of the tensegrity (difficult for complicated structures); and (2) to condense in a matrix the information about the tensegrity (using tools from symbolic computation).

Back to Main List

Stability conditions for tensegrity structures.

Zhang, Ohsaki,
Internat. J. Solids Structures 44 (2007), no. 11-12, 3875–3886.
Back to Main List

2. Highlights

Connelly 1980 If a tensegrity framework $G(\mathbf{p})$ is infinitesimally rigid in R^d, then it is rigid in R^d.

Connelly 1980 A tensegrity framework $G(\mathbf{p})$ is infinitesimally rigid in R^d iff it is statically rigid.
(Every equilibrium load can be resolved by a proper stress.)

Roth and Whiteley, 1981 Let $G(\mathbf{p})$ be a tensegrity framework in R^d. Then $G(\mathbf{p})$ is statically rigid in R^d if and only if there is a proper self-stress that is non-zero on each cable and strut, and $G'(\mathbf{p})$ is statically rigid, where G' is obtained from G by replacing each member with a bar.

Connelly, 1982 Let $G(\mathbf{p})$ be a tensegrity framework in the plane where the vertices form a convex polygon, all the external edges are cables, struts are the only other members, and $G(\mathbf{p})$ has a proper non-zero self-stress. Then $G(mathbf p)$ is globally rigid in R^d for all $d \geq 2$.

Connelly, 1982 If a tensegrity framework $G(\mathbf{p})$ is rigid in R^d and G has at least one cable or strut, then $G(\mathbf{p})$ has a proper non-zero self-stress.

Crapo, Whiteley, 1982 Plane stresses and projective polyhedra

Connelly and Whiteley, 1990 Let $G(\mathbf{p})$ be a rigid tensegrity framework in the plane, where the vertices form a convex polygon, all the external edges are bars, and cables are the only other members, then $G(\mathbf{p})$ is infinitesimally rigid in the plane.

Connelly and Whiteley, 1996

1st order rigidity

$\Downarrow$

Prestress stability

$\Downarrow$

2nd order rigidity

$\Downarrow$

rigidity

for any framework.

Examples show that none of these implications are reversible.

The Stress test.

3. Historic Questions

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