

# Problem A: Modeling National Debt

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## Problem Restatement

In recent history, the national debt of America has continuously increased due to imbalances between federal revenue and federal expenditures. In the future, the current youth of America will be forced to manage this increasing national debt and devise a system to pay it off or prevent its growth. By analyzing the relationship between debt rates, federal revenue, and federal expenditure, the economic situation of the future can be forecasted fairly well. The first objective is to create a model that can forecast the economic situation in the future based on parameters input by users. Using this model, several financial plans will be analyzed and a letter will be drafted to the President elect explaining this model. The letter will explain the benefits and shortcomings of financial plans and how they will affect the national debt in the future.

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## Assumptions and Justifications

- All national debt can be viewed as one loan with a constant interest rate which is continuously compounded every year. If all the vehicles of interest were taken into consideration, the model would have been too complex to work because the national debt is owed in a multitude of ways and each way has various interest rates. Consequently, to make the model simpler and more user friendly, national debt was simply viewed as one loan whose interest rate is an average of all interest rates of all the vehicles of investment.
- When the user enters in parameters for a given fiscal year, they expect the output debt to be given in the dollar value of the same fiscal year (basically, they do not want to take inflation into account). This gives a better model for the user because when they see the national debt for future years, he or she will not need to take inflation into account because the value of the dollar will be the same for all of the debts. Consequently, it will be easier for the user to determine how much better or worse the federal government (efficiency) is doing in terms of national debt from year to year.
- All national debt is being dealt with in US dollars. This allows a simpler model because if the national debt was in multiple currencies (The major currencies of trading are the US Dollar, Euro, and Pound), then the exchange rates between the currencies would need to be accounted for. And the previous assumption about inflation would not apply.
- If the federal expenditures are greater than the federal income, national debt equal to the deficit is taken (the government will always take debt to pay for its expenditures). This is a fairly good assumption because most of the time, the government will take debt to pay for its expenditures. Very rarely the government will find another way to make money to pay for its costs.
- The government is not lending money to any other nations for interest. Although it seems obvious that if the government itself is in debt, then it probably will probably not lend money to other nations. However, right now the government in fact has countries in debt to itself and is making money off the interest of these debts.
- If there is surplus income (federal income  $>$  federal expenditures), then all the surplus is used to pay off the national debt. This assumption insures that the national debt will actually be paid off in the future years to come because whenever the federal government actually makes money, they will spend it all on reducing their national debt.

All income sources in the US are taxed. No companies or individuals hide any part of their income from the government i.e. nobody cheats the law. For example, there are no illegal immigrants and no goods are bought on the "black market". This ensures that the user can enter in a proper value for expected income from person and find an accurate expected income in personal taxes for the nation. If this assumption was not to be made, then the illegal immigrants in the US would skew the population model and the unreported income would need to be accounted for by the user when entering expected income per person.

- International business and outsourcing will not significantly affect the income received from corporations. This assumption is made to simplify the model because with President Obama as the new president, it is not known what will happen with international business and outsourcing. He is planning on reducing outsourcing but the extent of these reductions are not known. Therefore, to simplify the model it was assumed that nothing drastic was going to occur regarding corporation income.
- National debt can be paid at any time as long as interest is compounded (there are no time limits and deadlines for the US government to pay the debts). This simplifies the model because it assumes that the US will not need capital in a certain moment of time to pay off its bonds, securities, and other debt sources. However, without this assumption, this factor would be near impossible to compute because with the current fear in the stock/bond markets, it is unknown how much money the citizens would lend the government.
- The population in the United States follows the current exponential growth rate (see **Appendix B**) \*\*Only used in model 1. Eventually, the population will cap off and not follow the model any longer, but because this model only deals with the near future (30 years or so), this assumption is justified.

## Variable Definitions

All of the variables are defined in this section. If any variables are unclear in the other sections, please refer to this section to find their definition.

**modeldebt** = The debt of the current year (the debt of the year prior to the year being predicted)

**newdebt** = The debt of the year that you want to calculate (debt of the year being predicted)

**totalincome** = the sum total of all the income received from federal receipts   \*\* **Only used in Model 1**

**startyear** = The current year (the year prior to the year being predicted).

**individual** = The average tax income per person in the year that you want to predict (forecast value).

**expenditure** = The total expenditure of the federal government in the year that you want to predict (forecast value).

**corporate** = The total corporate tax income in the year that you want to predict (forecast value).

**civil** = The total income from social insurance and retirement receipts (given to the government by citizens) in the year that you want to calculate (forecast value).

**otherincome** = The total income from all other federal receipts (forecast value).

**interest** = The average interest rate of the debt (forecast value).

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## Model Creation / Process

### ■ Background Research

Before the model was created, economic topics relating to national debt were studied. Understanding of the question and the raw data tables provided by sources made more sense after this background research. Topics studied include GDP, NNI, GNP, etc. See **Appendix A** for more detail.

### ■ Model 1 (Simple, first model created)

This model was created before any research or any data collection began. It was created to be expanded upon, and it helped to understand the problem because the model was so simplistic that it required minimal work. It did correctly calculate the new debt but the parameters were extremely broad. The model simply predicted the next year's debt based on the current year's debt, projected expenditures, projected income, and projected interest rates.

$$\text{newdebt} = (\text{modeldebt} * \text{interest}) + \text{expenditure} - \text{totalincome}$$

### ■ Model 2 (Personal income determined by entire population model)

Our process began by collecting data and conducting research. After identifying the variables and the magnitude of the effect of the variables on national debt, we were able to identify the optimal variables for our model. We chose to group all expenditures under one category because we did not model individual aspects of expenditure. The addition of all expenditures was justified because it allowed more accessible manipulation and ease of use for the model.

The group decided to build a recursive model based on the national debt from the previous year. We decided to do this because a recursive model would allow the variables to be changed on a year to year basis as opposed to a closed form model. The model was built on a year to year basis because interest is compounded annually. In addition, the group decided that a recursive model would be the best because real world trends often cannot be fit to an equation. By using a recursive model, extreme world events are accounted for (such as a national catastrophe).

Initially, the group planned to build a model that accepted variables in the form of input. Using these variables, the national debt of the next year was calculated based on the national debt of the current year. The form of this model was similar to the setup of model 1; however, we accounted for additional income factors.

We chose to expand on the income factors, opposed to the expenditure factors, because of our previous research. We found that the expenditure tends to increase with year, while income does not.

One factor that we found very important was the receipt that the government collects from each citizen. The total amount of money (in billions) that the government collects is not an accessible number. In addition, there is not a clear relationship in terms of population. To account for this situation, we modelled the receipt that the government collects from each citizen population by modelling the product of the income per citizen and the population at any given year. In order to make this sub-model, we needed to make a model of the population.

$$\text{newdebt} = \text{modeldebt} * \left(1 + \frac{\text{interest}}{100}\right) + (\text{expenditure}) - \left(-4.547 * 10^7 + 4.068 * 10^7 * 1.01^{\text{startyear}-1790} * \text{individual} + \text{corporate} + \text{civil} + \text{otherIncome}\right)$$

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## Strengths and Weaknesses of This Model

### ■ Model 1

#### Strengths

1) This is a very conceptually simplistic model.

#### Weaknesses

1) The parameters were extremely broad and required the user to do a lot of work before they could use the model.

2) The model was not based on realistic trends (such as population)

3) It was not flexible and could only be applied to simple situations.

4) The model made extreme assumptions (such as income is not based on population, tax rates, sales tax, excise tax, etc.).

### ■ Model 2

#### Strengths

1) Takes into account population growth (this is only a strength for short term).

2) It is more practical than the first model .

3) The inputs can be changed on a yearly basis.

4) Every input can be defined by its own function.

#### Weaknesses

1) Predictions become inaccurate after around 50 years (however, this model was only designed for a 20 year period).

2) The functions defining the variables may not perfectly model the data.

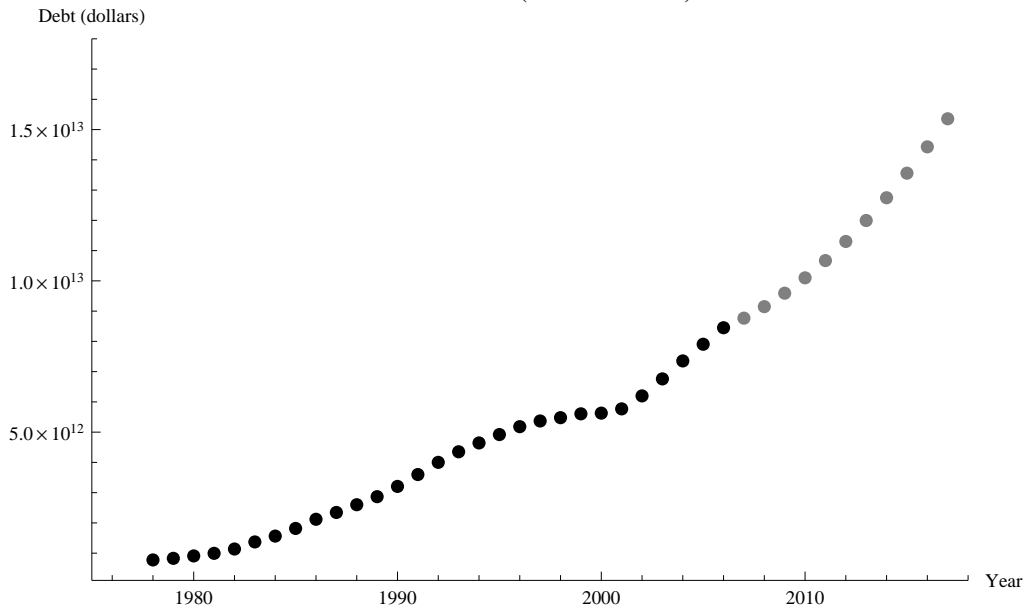
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## Model Testing and Redesign

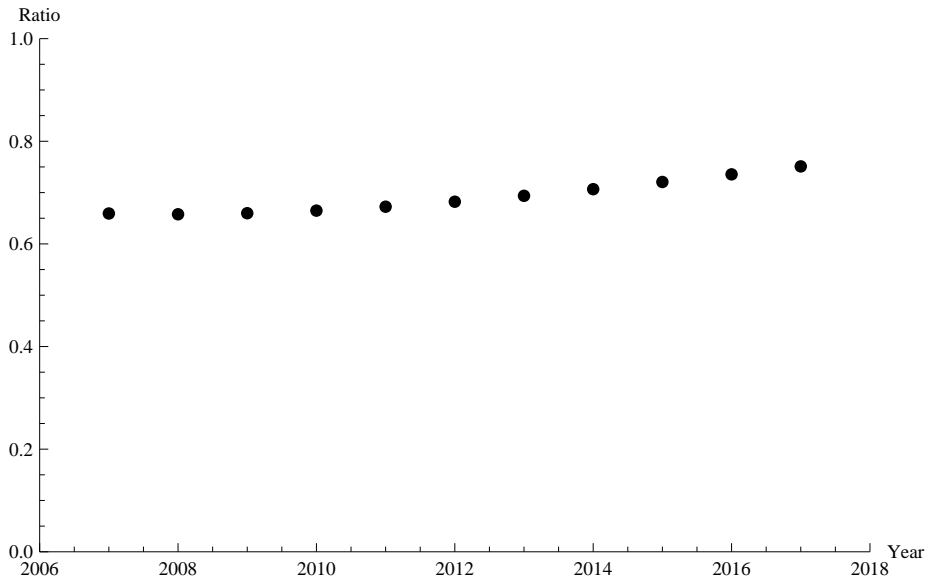
### ■ Financial Plan 1

Calculated using model 2 with values of Appendix J

Debt vs. Time (with Prediction)



Ratio of Debt to GDP vs. Time

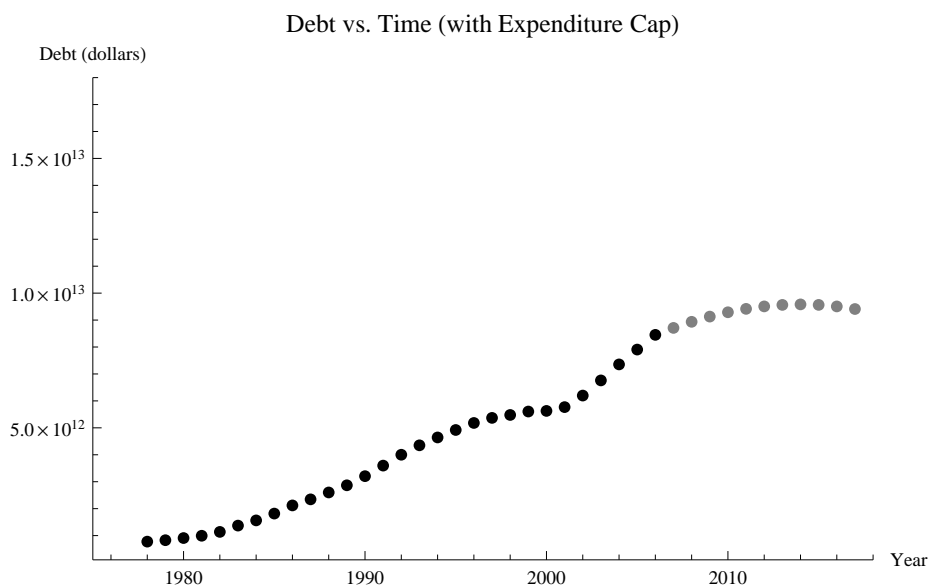


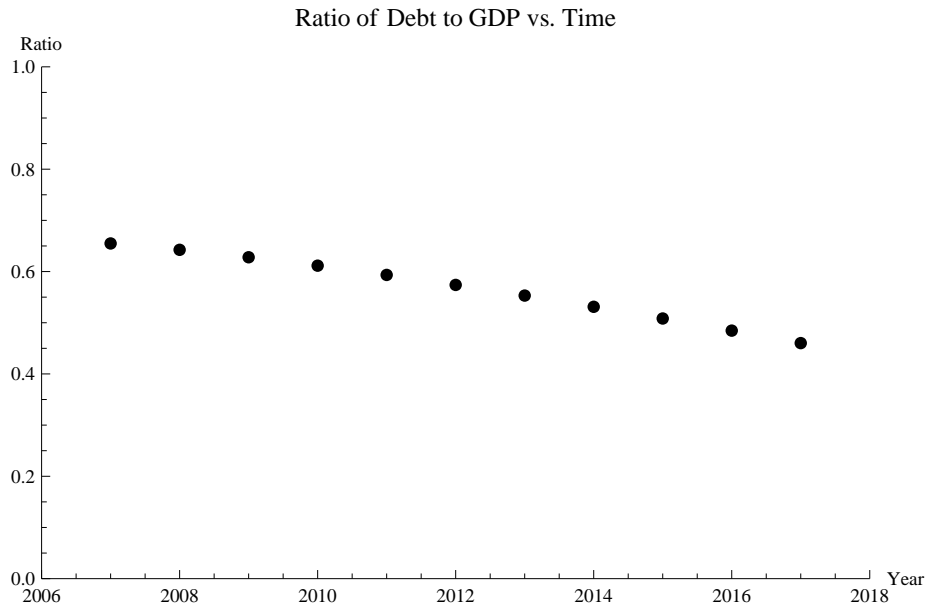
### ■ Analyzing Financial plan 1

If the current financial plan is followed from 2009-2017, the national debt will increase at an increasing rate. It can be seen from Figure 2 that the GDP growth will not be able to match this national debt growth and the ratio of national debt : GDP will increase to 0.75103 (the national debt will be 75.103% of the value of the nation). Therefore, this financial plan will have severe detrimental effects on the national economy. It can be extrapolated that if this trend continues until 2040, the national debt will be greater than the GDP. Basically, the debt we owe to other nations will be greater than the total value of our nation. This analysis proves that the current financial plan of the US needs to be severely reformed in order to ensure our nation does not become bankrupt.

### ■ Financial Plan 2

Calculated using model 2 with values of Appendix K





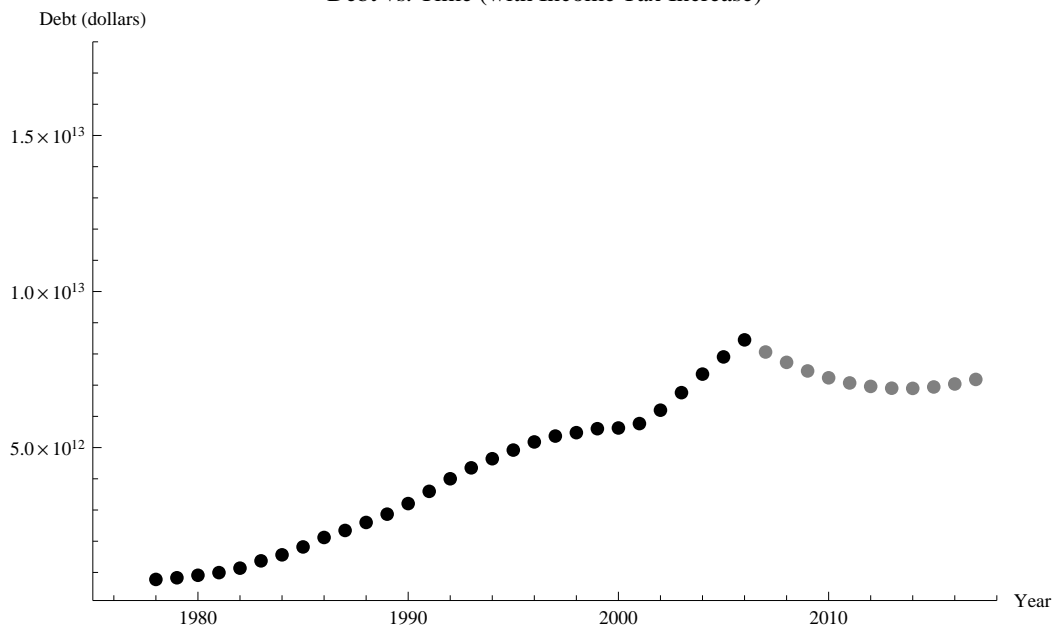
### ■ Analyzing Financial plan 2

In this model, all the inputs are based on trends of current financial activities, the only difference being that the expenditure increases by no more than 50 billion dollars a year. Although the debt will rise from 2009 to 2014, after 2014 the national debt will actually begin to decrease. In addition, the national debt to GDP ratio will always decrease, regardless of the year (even when national debt is increasing before 2014). Although this seems counter-intuitive, it makes sense when the federal income per year is analyzed. As the year increases, the population model shows that the populace increases exponentially. Therefore, if the average income tax paid per person remains constant and the other tax returns follow current trends, then the income of the nation will increase. After a certain point the income will outweigh the expenditure because of the taxes placed on the increasing population. Therefore, the debt would began to decrease at an increasing rate after 2014. This is our best plan because it gives a slow but permanent solution to the national debt problem while allowing expenditures to increase at a viable rate.

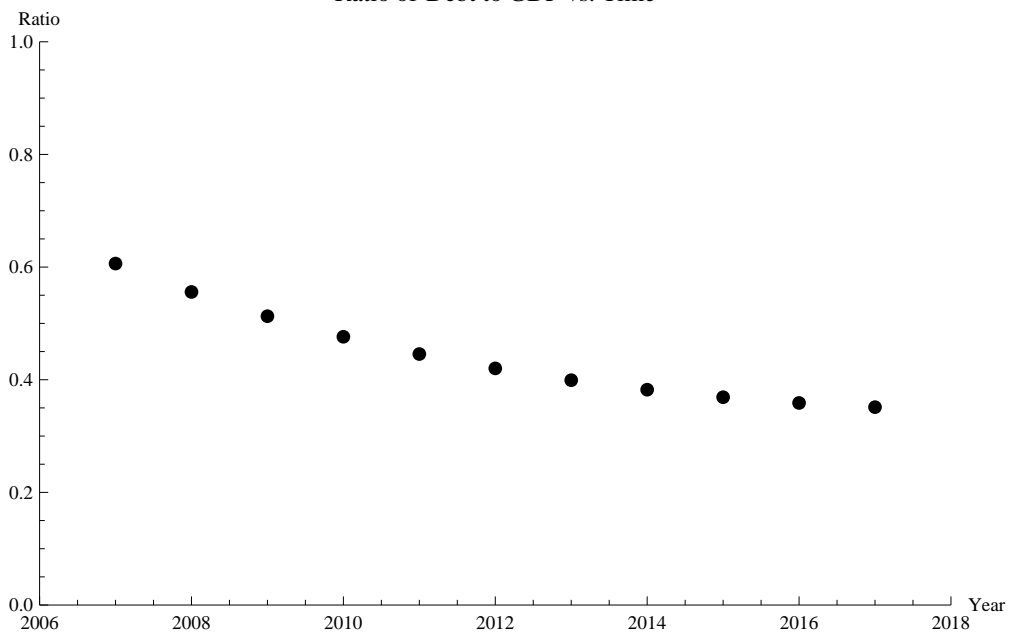
### ■ Financial Plan 3

Calculated using model 2 with values of Appendix L

Debt vs. Time (with Income Tax Increase)



Ratio of Debt to GDP vs. Time



### ■ Analyzing Financial plan 3

In this model, all the inputs are based on trends of current financial activities, the only difference being that the average money received per person increases permanently at the expected rate of \$2,000 per year. Although the debt will fall from 2009 to 2015, after 2015 the national debt will actually begin to rise. Although, the national debt to GDP ratio will initially decrease, after 25 years, the ratio will start to rise again. Like financial plan 2, this seems counter-intuitive but it makes sense when the federal expense is analyzed. As with financial plan 2, as the year increases, the population model shows that the populace increases exponentially. Therefore, if the average income tax paid per person remains constant and the other tax returns follow current trends, then the income of the nation will increase. However, at the same time, the expenditure is increasing at an alarming rate (following current trends). Consequently, after a certain point the expenditures will outweigh the income because of the expenditure rate increasing. Therefore, the debt would begin to increase at an increasing rate after 2015. This is a bad plan because it increases taxes on the already struggling population and still results in greater national debt.

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## Letter to President Obama

Dear President Obama,

We are a team of 4 students in 11th grade at the Massachusetts Academy of Mathematics and Science at WPI. We have explored the relationship between national economic factors and the national debt for a recent Mathematics project. The national debt is of great importance to us because we, the future generation, will have to manage it in the future. Our team has researched factors that affect federal receipts and federal expenditures, trends in these factors, and trends in the size of the U.S. population. We have succeeded in modelling these factors and we have combined them to form a model for national debt. The model that we have developed is shown below in the section labelled "Model". Using our model and analysis of the trends in federal receipts and federal expenditures, we found that if the current financial plan set by President Bush continues to be followed, by 2017 the national debt will be over 75% of the GDP, which is the total value of the nation, and by 2040 the national debt will be greater than the GDP. From this analysis, it is clear that the current financial plan needs major reforms. In an effort to find how the financial plan should be reformed, we modelled two additional plans; in the first one, the increase in federal expenditure per annum was limited to 50 billion dollars and in the second one, the average tax collected per person was increased by \$2000 per annum. Surprisingly, the first financial plan reduced national debt more effectively than the second. The first financial plan which is extremely feasible for it allows additional expenditure a year (it just limits the extent of this expenditure) resulted in higher income because the population increases as the year increases leading to higher tax revenues. Although, in the second situation (when tax rates were increased), the national debt dropped initially, while over a long period of time the national debt increased. This is because the expenditure eventually outweighed the income received from the extra taxes. Also, this financial plan is not very feasible because it will increase taxes on the already struggling American families. We urge you to use our debt model and our first financial plan to help solve the national debt crisis in the U.S.

## Model

$$\text{newdebt} = \text{modeldebt} * \left(1 + \frac{\text{interest}}{100}\right) + (\text{expenditure}) - \left(-4.547 * 10^7 + 4.068 * 10^7 * 1.01^{\text{startyear}-1790} * \text{individual} + \text{corporate} + \text{civil} + \text{otherIncome}\right)$$

modeldebt = The debt of the current year (the debt of the year prior to the year being predicted)

newdebt = The debt of the year that you want to calculate (debt of the year being predicted)

startyear = The current year (the year prior to the year being predicted).

individual = The average tax income per person in the year that you want to predict (forecast value).

expenditure = The total expenditure of the federal government in the year that you want to predict (forecast value).

corporate = The total corporate tax income in the year that you want to predict (forecast value).

civil = The total income from social insurance and retirement receipts

(given to the government by citizens) in the year that you want to calculate (forecast value).

otherincome = The total income from all other federal receipts (forecast value).

interest = The average interest rate of the debt (forecast value).

Sincerely,

Team 1974 of the HiMCM Competition

## Appendices

### ■ Appendix A: Background Research

#### What is GDP?

GDP (Gross Domestic Product) is the total market value of all the final goods and services produced within a nation in a given period of time. It is often viewed as the "worth" of a country and is one of the primary indicators used to gauge the health of a nation's economy. GDP is expressed either as a dollar value or as a comparison to the previous quarter/year. For example, this year's GDP can be expressed as 300 billion dollars or it could be expressed as "up 3%" (means that the economy has grown by 3% over the last year/quarter). The GDP can be calculated by using the equation shown below:

$$\text{GDP} = C + I + G + (X - M)$$

C = Private Consumption in the Economy. This includes personal expenditures of households like food, rent, medical expenses, etc. However, it does not include the expense of new housing (People buying new houses).

I = Investments by business or households in capital. This includes purchases of new land (mines, oil drilling rights, etc.),

purchase of machinery and equipment, and any other investment that results in instant capital. In addition to this business investment, the purchase of property by all people is also included in this. However, it does not include purchase of financial products such as bonds and stocks.

$G$  = Government expenditures on final goods and services. It includes salaries of government workers, purchase of military weapons, and any other investment expenditure of the Government. However, it does not include transfer payments (a redistribution of wealth in the nation) such as social security, welfare, and unemployment benefits.

$X$  = Gross exports of a nation. This is the value of goods and services produced and sold to other nations.

$M$  = Gross imports of a nation. These are subtracted because it is included in  $G$ ,  $I$ , and  $C$ . It must be deducted to avoid counting foreign supply as domestic.

What is GDP Per Capita?

GDP per capita is defined as the Gross Domestic Product per person. This is the mean income of a person in a country.

Limits

GDP per capita is often used as an indicator of standard of living. However, it does not include wealth distribution, voluntary work, what is being produced, the Quality of goods, Susceptibility of Growth, and the Black Market (where money spent is not recorded)

### What is GNP?

GNP measures the total output of all the nationals of a specific country. It is different from GDP for it deals with groups instead of regions. However, with the emergence of a global economy, the GNP is being used less and the GDP is being used more because the nationals of a country are now spending their money in the country they reside in, not in their country of origin.

$GNP = GDP + \text{income earned by citizens abroad} - \text{income earned by foreigners in the country}$

### What is NDP?

NDP is the Net Domestic Product and it can be calculated by subtracting the depreciation of capital goods from the GDP. The NDP can be used to estimate of how much a country must spend to maintain its current GDP. The Net Domestic Product takes into consideration the depreciation (decrease in value of a good as time increases), so it is more effective at calculating the value of a nation than the GDP. However, the depreciation is very hard to predict.

$NDP = GDP - \text{depreciation}$

### What is National Debt?

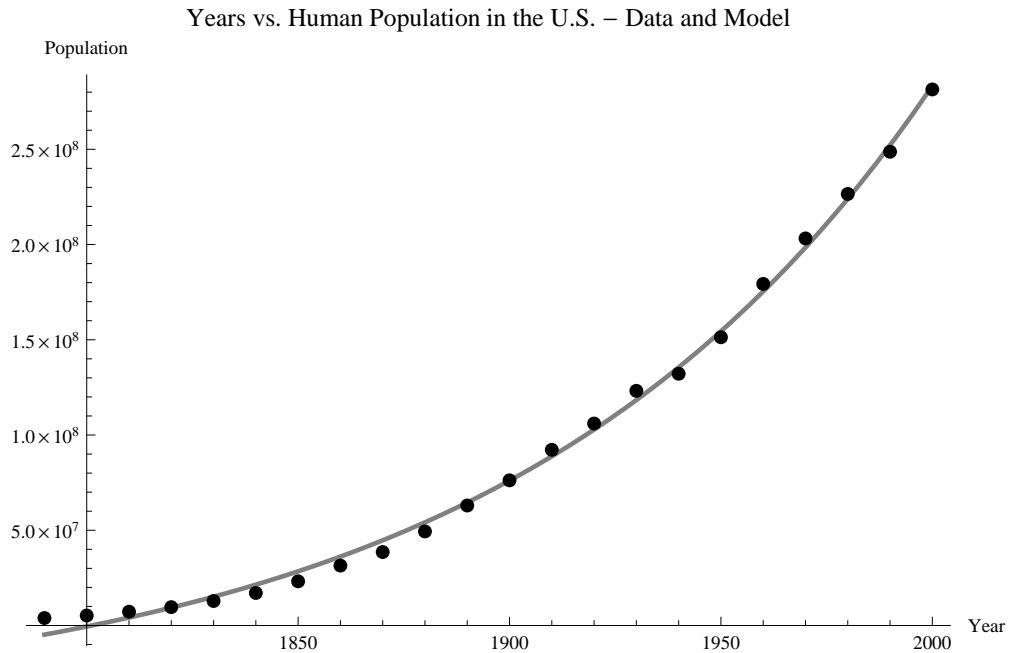
Government debt can be seen as indirect debt (money owed to lenders in the country) + the external debt (money owed to foreign lenders). Governments can borrow money by issuing securities, bonds, bills, etc. The government must pay interest rates on the debts it takes; the interest rates vary according to debt vehicle and time.

### What is a Receipt?

Receipts are sums of money that have been received in change for goods and services. Federal government receipts are sources of federal income. These include income tax, corporation tax, excise tax, social services, etc.

## ■ Appendix B: Model of US population

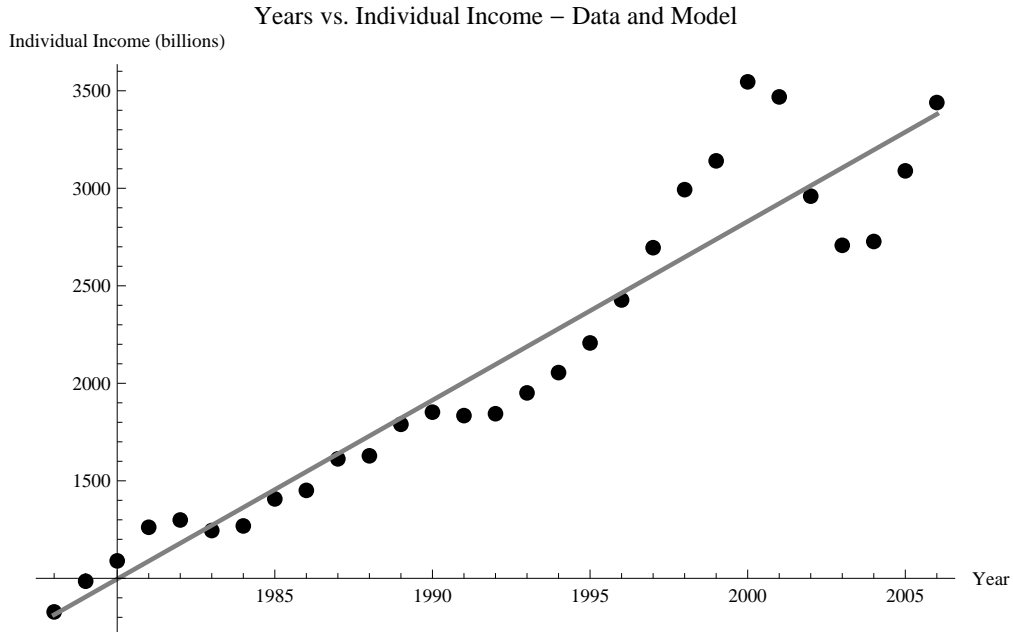
Data of United States population was gathered from wikipedia. A graph of this data was created and analyzed. A model of the population was created.



$$\text{population}[\text{year}] = -4.547497629814653 * 10^7 + 4.068170804222587 * 10^7 * 1.01^{\text{year}-1790}$$

## ■ Appendix C: Model of Individual Receipt

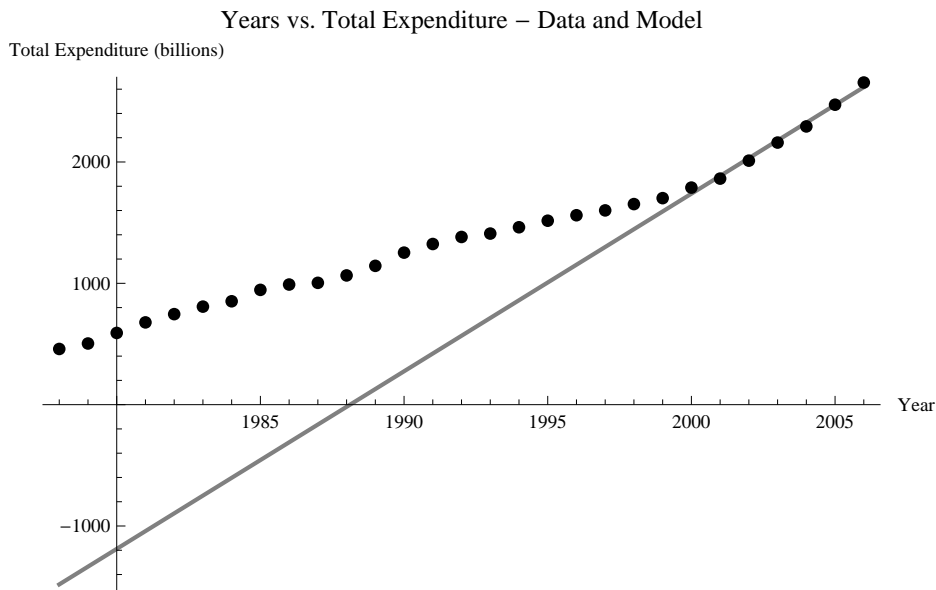
Using figures of the total receipt gained from individual citizen forces from the years 1978 to 2006, a model of the income per person in the United States was produced.



$$\text{individual}[\text{year}] = -180\,503.12878913368 + 91.66655204570132 * \text{year}$$

■ Appendix D: Model of Expenditure

Using figures of the total expenditure from the years 1978 to 2006, a model of the expenditure in the United States was produced. The model was optimized to forecast for the domain of 1999 to 2017; therefore, the preceding data (from year 1978 to year 1998) does not fit the model very well.



$$\text{expenditure}[\text{year}] = (-290\,975.7857142202 + 146.35714285710995 \text{ year}) * 1\,000\,000\,000$$

### ■ Appendix E: Model of Civil Receipt

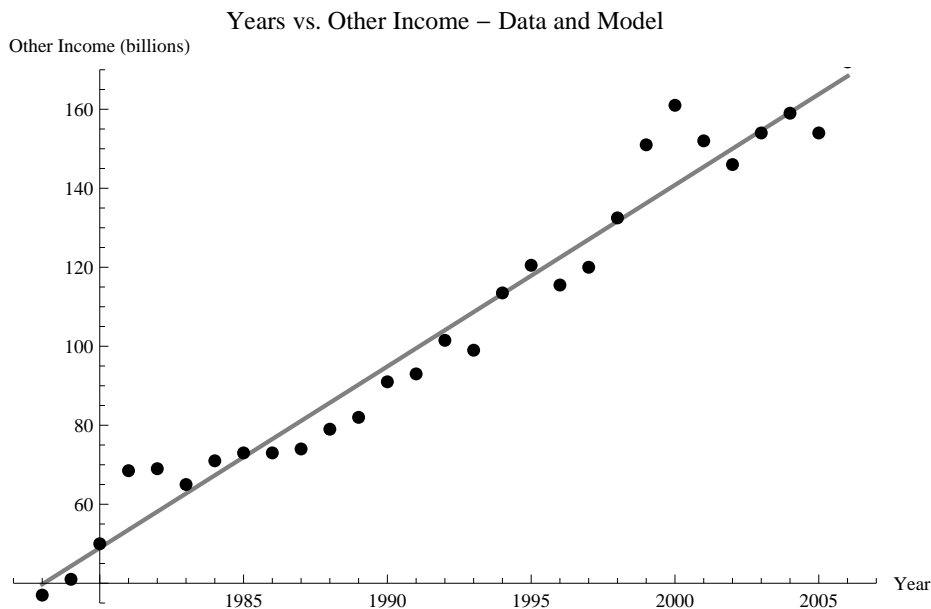
Using figures of the corporate receipts from the years 1978 to 2006, a model of the corporate receipt in the United States was produced.



$$\text{civil}[\text{year}] = (-14918.3 + 7.55 * \text{years}) * 1\,000\,000\,000$$

### ■ Appendix F: Model of All Other Incomes

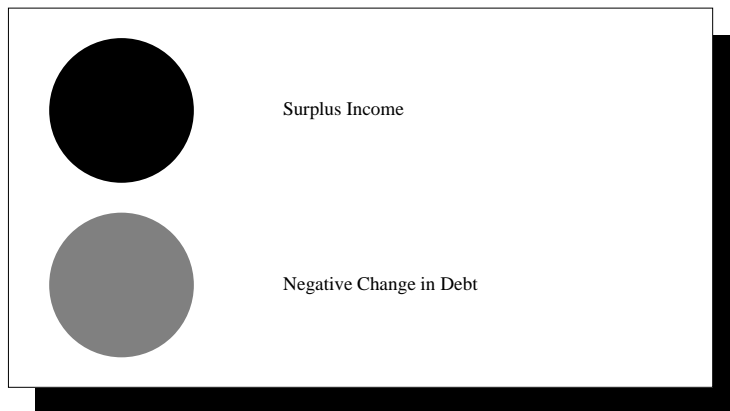
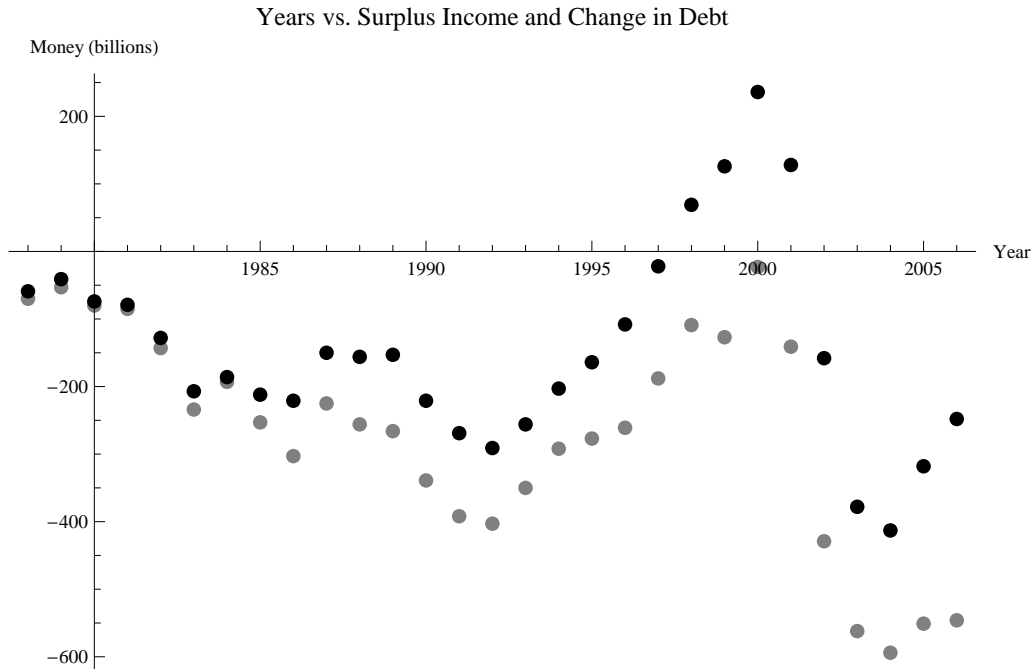
After identifying the most important variables that affect the current economic situations, we were left with all other types of incomes based on data from the United States Government from the year 1978 to 2006.



$$\text{otherincome} = (-9047.355665024783 + 4.594088669950814 * \text{year}) * 1\,000\,000\,000$$

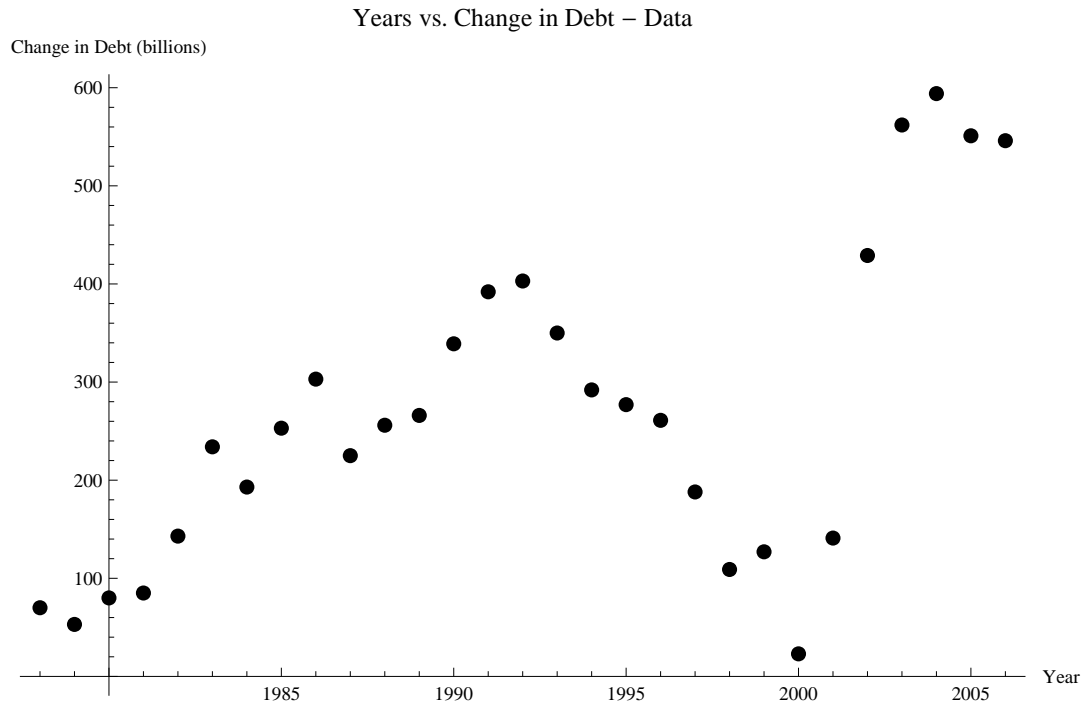
## Appendix G: Actual and Predicted debt

A comparison of surplus income and the change in debt was conducted to find the interest rate on the national debt of the previous year. It was predicated on the assumption that all surplus income is used to reduce national debt.



## ■ Appendix H: Change in Debt

Change of debt was calculated by finding the difference between the current debt and the debt of last year.



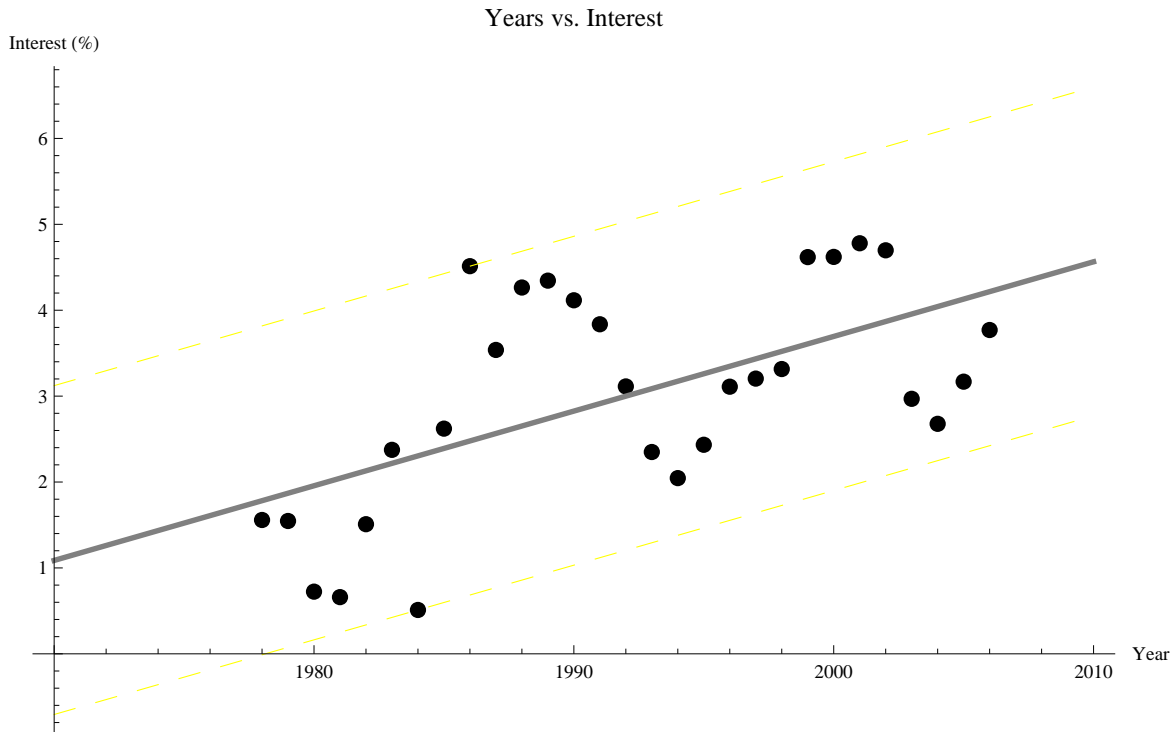
### ■ Appendix I: Model of Interest

After we graphed the actual and predicted debt, we were able to find interest by computing:

$$\frac{\text{expenditure} - \text{receipt} - \text{deltadebt}}{\text{debt}_{\text{year}-1}}$$

for each year.

After, the percent of interest was modeled to be used to forecast future interests.



interest[years] =  $-1.70145 + 0.000869199 * \text{years}$

### ■ Appendix J: Definition of Variables for Plan 1

modeldebt = 845100000000

individual[year] =  $-180503. + 91.6666 \text{ year}$

expenditure[year] =  $(-290976. + 146.357 \text{ year}) * 1000000000$

corporate[year] =  $(-14918.3 + 7.55357 \text{ year}) * 1000000000$

civil[year] =  $(-49187.3 + 24.9126 \text{ year}) * 1000000000$

otherIncome[year] =  $(-9047.36 + 4.59409 \text{ year}) * 1000000000$

interest[years] =  $-1.70145 + 0.000869199 * \text{years}$

### ■ Appendix K: Definition of Variables for Plan 2

modeldebt = 845100000000

individual[year] =  $-180503. + 91.6666 \text{ year}$

expenditure[year] =  $265500000000 + 50000000000 * (\text{year} - 2006)$

corporate[year] =  $(-14918.3 + 7.55357 \text{ year}) * 1000000000$

civil[year] =  $(-49187.3 + 24.9126 \text{ year}) * 1000000000$

otherIncome[year] =  $(-9047.36 + 4.59409 \text{ year}) * 1000000000$

interest[years] =  $-1.70145 + 0.000869199 * \text{years}$

## ■ Appendix L: Definition of Variables for Plan 3

modeldebt = 845100000000

individual[year] = **-180 503. + 91.6666 year + 2000**

expenditure[year] = (-290 976. + 146.357 year) \* 1 000 000 000

corporate[year] = (-14 918.3 + 7.55357 year) \* 1 000 000 000

civil[year] = (-49 187.3 + 24.9126 year) \* 1 000 000 000

otherIncome[year] = (-9047.36 + 4.59409 year) \* 1 000 000 000

interest[years] = -1.70145 + 0.000869199 \* years

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### Literature Reviewed (References)

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