Closing the Loop: Forging High-Quality Agile Virtual Enterprises in a Reverse Supply Chain via Solution Portfolios

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Abstract: Reverse supply chains are receiving increased attention for business and sustainability opportunities. As few organizations are adept at both forward and reverse supply chains, subcontracting various activities is imperative. Vendor selection that best achieves combined expertise for reverse supply chains, while quickly forming virtual enterprises to seize market opportunities, is an emerging issue. We formulate a novel 0-1 integer nonlinear optimization model, subsequently linearized to enable efficient computational performance, to select vendors that minimize the maximum formation time for creating agile virtual reverse supply chains. We then generate a portfolio of diverse, high-quality vendor assignments by adapting a recent algorithmic technique, thereby allowing industrial managers to address intangible factors into their final decisions. Computational experiments on simulated data demonstrate the model's efficiency for generating sets of high-quality and diverse solutions in reasonable timeframes.

Keywords: Virtual Enterprise; Reverse Supply Chain; Agile Enterprise; Sustainability; Integer Programming; Diversity

Introduction

Reverse logistics and supply chains have taken on a broader meaning over the past two decades. What used to be solely a business concern in managing product returns to manufacturers, reverse logistics has expanded to encompass an environmental sustainability dimension by seeking to extend the life of products and materials (Gunasekeran et al., 2014; Altekin et al, 2017; Zhu et al., 2008). The shift in reverse supply chains to 'close-the-loop' has necessitated organizations to incorporate reverse logistics into their supply chains (Jayaraman et al., 1999; Srivastava, 2008). Throughout the world, recent legislation and new policies such as the circular economy have heightened the importance and value of recycling of wastes and, specifically, electronic wastes (Li et al., 2014; Nowakowski, et al., 2018) and reuse of materials. Therefore, the interest in forging e-waste supply/value chains to seek potential profitability and grasp market opportunity has been growing rapidly in recent years. Because electronic products have very short life cycles, managing the e-wastes must be conducted in a timely fashion. As few organizations are competent at both forward and reverse supply chains, subcontracting various reverse logistics activities has become a common practice, and e-waste supply chains are no exception (Guarnieri, et al., 2015; Grabara and Kot, 2017).

As part of this subcontracting practice, organizations may seek out a single broker or a fourth party to support their reverse supply chain activities (Krumwiede and Sheu, 2002). This type of temporary organizational formation for the purposes of providing reverse supply chain services is similar to virtual organizations or virtual enterprises forming interim partnerships to take advantage of a short-term market opportunity (Dong and Wan, 2016; Sha and Che, 2006; Shamsuzzoha, et al., 2017). Our focus is on these virtual, temporary organizations that develop to deal with products having sensitive time value, such as electronic waste. Market sensitivity requires such organizations to be agile.

Agile virtual reverse supply chain (AVRSC) is formed by a number of independent, yet complementary organizations, selecting the right members with needed resources and technical expertise, in a timely fashion, is critical. To this end we introduce an optimization model to form an AVRSC to take on a promising and time-sensitive business opportunity such as the resale of end-of-use electronic products. The model sets the stage for identifying a portfolio of attractive groupings of providers that can form a virtual enterprise in a timely manner. The portfolio is guaranteed to include an optimal solution (if one exists), and the remaining optimal and near-optimal solutions incorporate diversity, as they are sequentially generated by simultaneously balancing the desire for solution *quality*, with solution *diversity*. The result is a diversified solution portfolio from which decision-makers can select a set of high-quality AVRSC provider partners. We consider the decision-maker to be an organization seeking to manage the formation of a virtual enterprise, such as a fourth-party reverse logistics provider who wishes to take advantage of a temporary market opportunity.

While the argument for optimal and near-optimal solutions within the context of optimization is clear, the introduction of multiple, diverse solutions is compelled by practical reasons. By providing a decision-maker with a set of diverse solutions, they can use their experience to evaluate alternative optimal and near-optimal solutions with respect to intangible factors such as proprietary issues, fairness and equity considerations, and other abstract factors that are difficult to quantify. Alternative optimal and near-optimal solutions that are overly similar are unable to offer the decision-maker such flexibility.

We make the following four major contributions in this paper. From a theoretical perspective, we first provide a novel integer nonlinear programming formulation with the goal of minimizing the maximum formation time, the solution of which identifies optimal AVRSC partners while simultaneously satisfy budget, quality, demand, and cycle time constraints. Second, we show how to adapt an earlier technique of Trapp and Konrad (2015) to identify a diverse set of high-quality (as characterized by the objective function evaluation) solutions for integer programming problems with *continuous variables*. As Trapp and Konrad (2015) consider only the binary case, the present study is the first to consider this adaptation. Our technique presents an effective decision-making method that, built upon the novel integer nonlinear programming formulation, sequentially generates diverse portfolio of optimal and near-optimal solution alternatives for virtual enterprise formation. Third, from an applications perspective, we explicitly consider time-sensitivity in our model, a critical aspect of successful AVRSCs. Lastly, although virtual reverse supply chains have been conceptualized for well over a decade (Browne and Zhang, 1999; Meade et al., 2007), the issues and decision-models associated with agile virtual enterprise formation has received little attention in the literature. To contribute and advance the literature in reverse supply chains as well as agile virtual enterprise formation, we investigate the nexus of these areas.

Within this context we initially provide background on the issues facing AVRSC in practice and research. The literature review summarizes existing approaches and identifies how the methodology introduced in this paper helps to fill an important gap in the literature. The integer nonlinear program is then presented and analyzed theoretically, including a reformulation to a mixed-integer *linear* program, followed by a brief introduction to the technique that we adapt to generate a portfolio of high-quality yet diverse solutions, numerical illustrations, computational experimentation and related discussions. The paper concludes with a summary of general observations, limitations associated with the study and directions for future research.

Literature Review

We classify the relevant literature into three categories. The first category defines the general structure of a reverse supply chain. Clearly identifying the structure and activities provides a practical foundation for formulating the decision-making problem under study. The second category discusses vendor selection and virtual enterprise formation techniques. The third category has to do with the solution portfolio approach, which aims to generate a set of optimal and near-optimal, yet distinctive, vendor selection solutions.

General Structure of a Reverse Supply Chain

Blackburn et al. (2004) identify the following five key processes in a reverse supply chain:

(1) *Product acquisition*: obtaining the used product from the user;
(2) *Reverse logistics*: transporting the products to a facility for inspecting, sorting, and disposition;
(3) *Inspection and disposition*: assessing the condition of the return and making the most profitable decision for reuse;
(4) *Remanufacturing/Refurbishing*: returning the product to original specification;
(5) *Marketing*: creating secondary markets for the recovered products.

It is not difficult to envision that the complexity of each process is determined by the composition of a product or material and that each process may need to be completed by a separate organization with specialized resources and expertise. Furthermore, other factors such as geographical locations, the number of possible vendors with suitable capabilities and capacities, and the volume and quantity of returned items will inevitably complicate the formation and operations of each reverse supply chain (e.g., Brandenburg and Rebs, 2015).

Major activities in a reverse supply chain network are shown in Figure 1 (Presley et al., 2007). At each stage potentially different organizations can be involved. The stages are interconnected with the sequence of events shown by arrows linking the activity boxes. These organizations may range from an original equipment manufacturer (OEM) to specialists in disassembling or transporting products and materials. Figure 1 is an illustrative graphic of a generic reverse supply chain. Specific stages and players will vary depending on the product, market, and purpose of the reverse supply chain. For example, a reverse supply chain for electronics remanufacturing will likely be different than a reverse supply chain for managing warranties associated with returned consumer products.

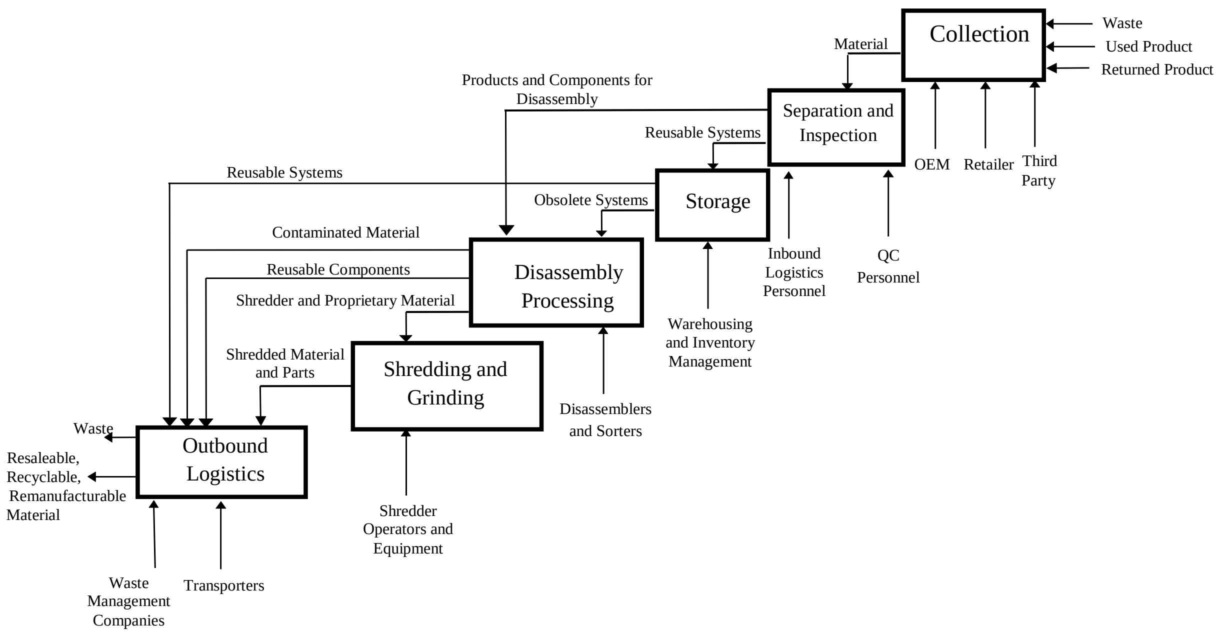

Figure 1: The Detailed Structure of a Reverse Supply Chain Network (Adapted from Presley et al., 2007)

Vendor Selection Techniques

Vendor selection is an essential aspect in forging collaborative partnerships in strategic sourcing as well as an important factor in the partnership's ability to successfully complete daily operations. As such, this subject has received wide attention and has been studied extensively in the contexts of both forward supply chain network formation (de Boer et al., 2001; Ding et al., 2015; Govindan et al., 2013; Igarashi et al., 2013; Simic et al., 2017), as well as closed-loop supply chain configuration (e.g., Brandenburg and Rebs, 2015). Typically the supplier selection decision is made under the premise that there exists a set of criteria, the importance of each criterion, and a pool of vendors with performance attributes or operational parameters, which are collectively referred to as discrete alternative multi-criteria decision-making. Depending on the complexity and the strategic importance of the decision-making situation, the techniques range from a straightforward weighted linear sum approach to more sophisticated stochastic modeling.

We group the existing selection methods into the following three categories: (1) *Ranking-based*: These techniques aim to derive or calculate a score for each vendor candidate based on a given set of criteria and their levels of importance. When the scores for all vendors under consideration are obtained, they can be ranked in descending order indicating the best choice. Several well-known methods fall into this category such as Total Cost of Ownership (TCO), Supplier Scorecard, Analytic Hierarchy Process (AHP), and the extended version of AHP – Analytic Network Process (ANP) (Asadabadi, 2017; de Boer et al., 2001; Meade and Sarkis, 1998; Sarkis et al., 2007; Verdecho et al., 2012). Some more sophisticated methods in this category integrate not only qualitative and quantitative factors into decision-making, but also consider uncertainty. Such techniques include Outranking Methods (Pirlot, 1997; de Boer et al., 1998) and Multi-Attribute Utility Theory (MAUT) (Shaik and Abdul-Kader, 2011) and various fuzzy set approaches (Simic et al., 2017). Additionally, different types of data and restrictions on the choices of weight values can also complicate the ranking method (e.g., Farzipoor 2009). (2) *Deterministic optimization-based*: These methods specify a managerial objective subject to a set of clearly defined constraints, which are formulated into a mathematical model with the selection choice as decision variables. Examples within this category include integer programming formulations (Glickman and White, 2008; Trapp and Sarkis, 2016) and data envelopment analysis (Ding et al., 2015; Liu et al., 2000). When there are multiple objectives to consider, the optimization problem may be formulated as a goal program. (3) *Stochastic modeling-based*: To capture the dynamic and uncertain elements inherent in the supplier selection decision-making process, a variety of techniques from operations research, and computational and mathematical sciences, can be used. For example, simulation (Ding et al., 2005), fuzzy logic (Jindal and Sangwan, 2016; Simic et al., 2017), expert systems (Yigin et al., 2007), artificial intelligence (de Boer et al., 2001), and genetic algorithms (He et al., 2009) have been employed in supplier selection. In some cases, combinations of several methods are used not only to identify qualified suppliers, but also to accommodate diverse procurement situations (Ho et al., 2010).

The methodology proposed in this study is best classified as deterministic optimization. This methodology, further described in the next section, seeks to determine high-quality and yet structurally diverse solutions which select vendor groups. The modeling approach has advantages and disadvantages similar to other mathematical programming formulations involving multiple criteria, such as ease of completion sensitivity analyses and moderate to low difficulty of adoption and data requirements. While our methodology may be somewhat more sophisticated for management to understand than simple scoring or multi-attribute techniques, the results of our methodology present a practical alternative to evaluate alternative solutions. Although the methodology introduced in this paper integrates economic factors (e.g., a budgetary constraint), its major focus is on helping management decision makers quickly arrive at a high-quality solution that will help them form a partnership of vendors in a timely fashion to service a market need and capture a market opportunity. Hence, the primary modeling emphasis is on formation time, rather than an economic objective typical of many other analytical models.

Solution Portfolios with Applications to Vendor Selection

Several studies (Tsai et al. 2008; Danna and Woodruff, 2009; Schittekat and Sörensen 2009; Camm, 2014) discuss the merit of having multiple solutions of "good" quality for the benefit of decision makers. Portfolio solutions that incorporate diversity are further beneficial; two solutions that, while technically distinct, are actually very similar, may not be of much value to the decision maker. Recently Trapp and Konrad (2015) discussed a method to sequentially generate a portfolio of diverse optimal and near-optimal solutions to binary integer programs. The technique begins by identifying a single optimal solution to the original optimization model (assuming one exists), and then proceeds to maximize the (normalized) ratio of *diversity*, as expressed by distance from the centroid of solutions in the portfolio, to the *loss in objective function quality*. The technique has been used to generate a diverse portfolio of solutions to a supplier selection problem in Trapp and Sarkis (2016). Moreover, Petit and Trapp (2015) adapt the approach to find diverse solutions of high-quality to combinatorial problems using constraint programming techniques, and Petit and Trapp (2019) extend these ideas further by developing and demonstrating a framework to infuse solutions to combinatorial problems with ad-hoc quality notions.

Vendor selection is an important prerequisite for the formation of virtual enterprises (Meade et al., 1997). While a number of vendor selection techniques for forging AVRSC enterprises exist, they largely fail to address two major decision factors. First, as AVRSC is very sensitive to time, the speed of selection and formation is critical and the time until each vendor commits to the AVRSC must be considered. Formation time is critical for responding in an agile manner to short-term, limited-time market opportunities. Organizations need to develop agile practices that will allow for this rapid partnership formation, which is an important way to gain a competitive advantage for these short term market opportunities (Yusuf et al., 1999). As the time to market products needs to be reduced in an opportunistic market situation, the selection of partners and formation of the virtual enterprise needs to be rapid (Goldman, 1995; Gunasekaran, 1998). Agile tools that can do this include information technology, organizational process adjustment, advanced manufacturing technology, advanced human resource practices, building trust and flexible contractual relationships (Gunasekaran, 1998; Nejatian and Zarei, 2013). Although we are not investigating the specific enablers for rapid formation of agile virtual enterprises, it can be assumed that organizations wishing to be considered as an AVRSC participant will adopt some of these agile virtual enterprise practices. Also, the individual time formation capabilities will be different depending on how much a specific organization has invested in these enablers.

Second, decision-makers' preferences may have some flexibility in this situation, and thus it may be attractive to have some choice among high-quality solutions. Consequently, a selection method allowing the decision-maker to choose from a diverse set of optimal and near-optimal solutions within a reasonable timeframe is of great value.

This paper bridges this gap through a new optimization model that forges an efficient AVRSC, coupled with the solution method to find diverse solutions of high quality. The integer nonlinear optimization approach introduced in this paper, while bearing some basic similarities to other mathematical programming approaches, encompasses additional complexity. In particular, the method of Trapp and Konrad (2015) has until now been demonstrated solely in the context of binary integer optimization problems. The present work is the first to adapt their approach outside of this domain, applying it to mixed integer programs, that is, binary integer programs with continuous variables. We then examine how this adaptation could be applied in agile virtual reverse supply chains.

The Analytical Model and Methodology

In this section we propose a novel optimization model together with an adaptation to a recently developed solution portfolio approach (Trapp and Konrad, 2015) to help identify a portfolio of AVRSC partners. We illustrate the decision-making process in the context of end-of-use mobile phones. This context sets the practical foundations for our methodology. An initial model is presented that includes a description of notation and model formulation, followed by transformations to make the model more tractable for solution. The technique to produce a portfolio of diverse and high-quality AVRSC solutions, based on a reformulated integer linear programming (MIP) formulation, is then presented.

The Decision-Making Context

We use the reverse supply chains of mobile phones as a basis to depict the problem considered in this paper. The extremely short life cycles and rapid advent of new technologies are placing end-of-use, mobile phones at the forefront of reverse supply chain implementations (Franke, et al., 2006; Geyer and Blass, 2010). Original equipment manufacturers (OEMs) such as Motorola, Samsung and Apple, and network service providers such as AT&T, Verizon, and T-Mobile are actively taking back end-of-use handsets as a service to customers as part of their corporate environmental responsibility program, or for compliance reasons. Both OEMs and service carriers typically outsource the operations of reverse supply chains of phones to third-party enterprises

(Geyer and Blass, 2010); for example, ReCellular, PaceButler, and International Recycling Network (IRN) in the U.S. have identified the collection of end-of-use mobile phones as a business opportunity. Apart from alliances with OEMs and network providers, these take-back enterprises team up with non-profit organizations and retailers to access the stock of retired handsets (e.g., Guide Jr. et al., 2005).

The structure of a mobile phone reverse supply chain is similar to the general network illustrated in Figure 1 but has its own features, required processes and activities. Based on a comprehensive study (Neira et al., 2006), the major processes and activities of a reverse supply chain of end-of-use mobile phones is shown in Figure 2. It can be observed that the entire chain involves multiple stakeholders and each process contains complex activities that require specialized resources, capabilities and technologies. Furthermore, multiple players may exist at each process; for example, the collection step has a range of participants, including OEMs, network service providers, retailers, various collectors such as web-based collectors, non-governmental organizations and charities, and municipalities. Consequently, a great deal of coordination is imperative even at the collection point alone.

Moreover, mobile phone resellers need to react in a timely manner given that depreciation timing and the value of phones may be relatively uncertain. If a reseller wishes to retain the significant value of the returned (end-of-use) phones the reverse logistics formation time for partners needs to be relatively rapid and efficient (Guide Jr. et al., 2005).

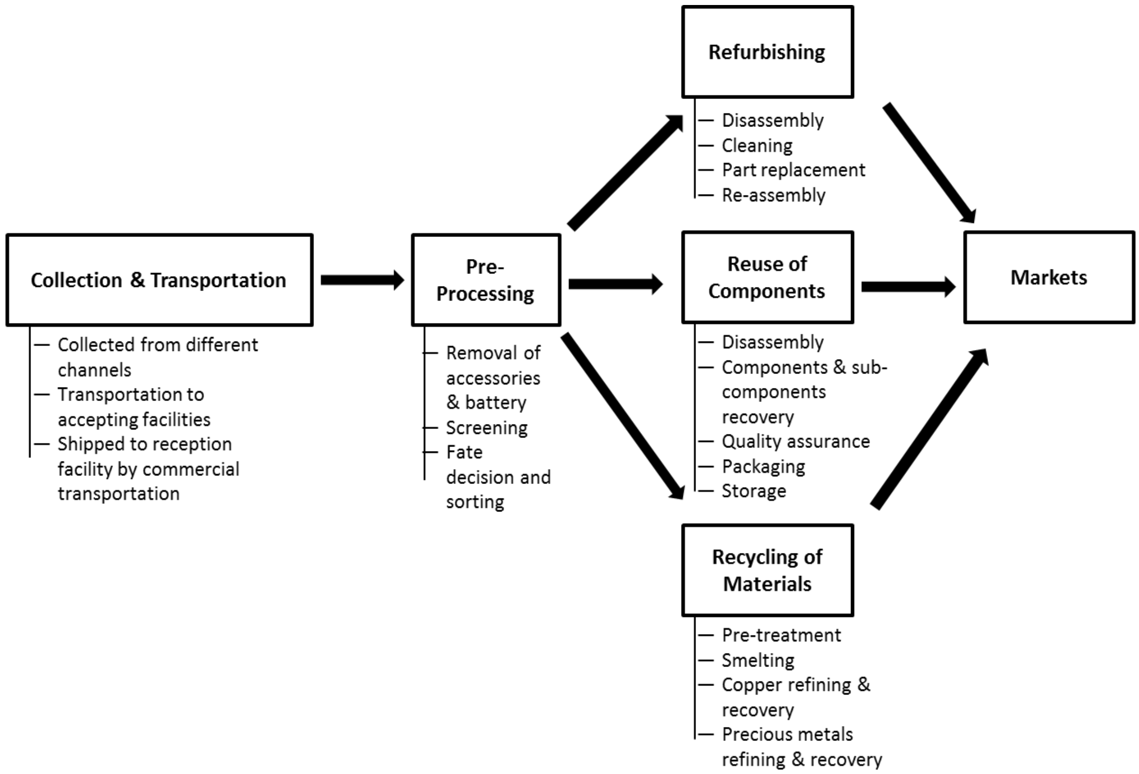

Figure 2: Reverse Supply Chain of Cell Phones (Adapted from Neira, et al, 2006)

Unfortunately, not all organizations are proficient at managing the entire mobile phone reverse supply chain, nor do they possess all the necessary expertise and resources. Therefore, to seize business opportunities as well as to protect the environment and promote sustainability, it is necessary to forge a virtual enterprise with carefully selected vendors/brokers at each stage to handle the entire end-of-use product supply chain. This situation represents a single product type environment. While some reverse supply chain organizations may be able to manage multiple products, thereby adding greater complexity to their management, the multi-product version is outside the scope of this paper, and we leave it as a direction for future research.

Agile Virtual Reverse Supply Chain (AVRSC) Problem Formulation

We now present the base optimization model to form optimal vendor team selections for a virtual reverse supply chain. Following the example of mobile-phone reverse supply chains, we consider forming a single-product virtual enterprise (such as PaceButler, specializing solely in mobile phones) and herein present our optimization model to select vendors to form such a single-product virtual reverse supply chain.

Definitions of Sets and Parameters

We first formalize the basic context described previously with mathematical notation. The problem environment considers that a number of reverse supply chain stages exist, denoted by set $\mathcal{S}$. The entire supply chain is geographically dispersed into multiple regions (set $\mathcal{R}$). The regional aspect may exist to help organizations develop more efficient processing of returned products or materials. The number of regions could be altered depending on the configuration of the AVRSC. Within each region (r) and stage (s) we assume there are one or more service providers ($\mathcal{P}_{rs}$) available. As in any supplier selection and evaluation approach, various business decisions, operations and supply chain strategic performance measures and parameters need to be considered. The modeling effort here explicitly includes the input parameters and performance metrics of investment cost or budget (B), cycle or delivery time (T), average quality (Q), and capacity – by satisfying demand (D). Specific parameters that contribute to these overall performance metrics are introduced for each provider in each region and stage.

While any of the above performance metrics may take priority, a crucial issue in (agile) virtual enterprise formation, to which we give precedence in our model below, is how quickly the

AVRSC can be formed. This criterion is especially critical if the marginal value of time of the product is high, such as in electronics. Thus, the responsiveness and effectiveness of the provider of a service for a specific stage in a region (formation time f_{rsp}) plays an especially important role in this model. Table 1 summarizes the definitions of sets and parameters used in our optimization model.

Table 1: Sets and Parameters Used in Mathematical Programming Formulation

Symbol	Definition
$\boldsymbol{\mathcal{S}}$	Set of stages (e.g., collection, sorting, storage, disassembly, reintegration), indexed by s
$\boldsymbol{\mathcal{R}}$	Set of regions (e.g., local, regional, national), indexed by r
$\boldsymbol{\mathcal{P}_{rs}}$	Set of providers for each region r and stage s, indexed by p
$\boldsymbol{B}$	Investment budget for the reverse supply chain formation process
$\boldsymbol{T}$	Threshold for total cycle time, over all stages, for completing reverse supply chain process
$\boldsymbol{D_{rs}}$	Demand for region r and stage s
$\boldsymbol{Q_{rs}}$	Quality threshold for completing reverse supply chain process in region r, stage s
$\boldsymbol{c_{rsp}}$	Cost of assigning region r, stage s to provider p
$\boldsymbol{a_{rsp}}$	The capacity available in region r, stage s for provider p
$\boldsymbol{q_{rsp}}$	Quality rating of region r, stage s for provider p
$\boldsymbol{t_{rsp}}$	Cycle time necessary to complete operations in region r, stage s for provider p
$\boldsymbol{f_{rsp}}$	Formation time necessary to "start up" operations in region r, stage s for provider p

Definition of Variables

Binary variables x_{rsp} indicate which stages and regions are assigned to specific providers:

$$x_{rsp} = \begin{cases} 1 & if\ region\ r, stage\ s\ is\ assigned\ to\ provider\ p; \\ 0 & otherwise. \end{cases} \tag{1}$$

Mathematical Programming Formulation

The problem of selecting vendors to forge a team of providers that minimize the maximum formation time of a virtual reverse supply chain is given as follows:

$$\textit{Minimize} \quad \max_{rsp}\{f_{rsp}x_{rsp}\} \tag{2}$$

$$\textit{subject to} \quad \sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}\sum_{p\in\mathcal{P}_{rs}} c_{rsp}x_{rsp} \leq B, \tag{3}$$

$$\sum_{p\in\mathcal{P}_{rs}} a_{rsp}x_{rsp} \geq D_{rs}, \qquad \forall\ r \in \mathcal{R}, s \in \mathcal{S}, \tag{4}$$

$$\sum_{p\in\mathcal{P}_{rs}} (q_{rsp} - Q_{rs})x_{rsp} \geq 0, \qquad \forall\ r \in \mathcal{R}, s \in \mathcal{S}, \tag{5}$$

$$\sum_{s\in\mathcal{S}} \max_{rp}\{t_{rsp}x_{rsp}\} \leq T, \tag{6}$$

$$x_{rsp} \in \{0,1\},\ \forall\ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}. \tag{7}$$

Objective (2) seeks to minimize the maximum formation time to assemble the virtual enterprise. Constraint (3) enforces budgetary limitations on the entities chosen in the virtual enterprise, while constraint sets (4) and (5) ensure that demand and average desired quality, respectively, are met in every region-stage. Constraint set (6) ensures that the total cycle (delivery) time over all stages does not exceed a certain threshold, T. Together, (2)–(7) constitute a binary integer formulation that is nonlinear due to the max expressions in both (2) and (6). Although (2)–(7) would likely be prohibitive to solve for realistic data instances, we next discuss how to reformulate these nonlinearities, making the resulting formulation amenable to solution with off-the-shelf MILP solvers such as CPLEX (IBM ILOG CPLEX, 2018).

Linearizing Formulation (2)–(7)

The *max* expressions in (2) and (6) of binary integer program (2)–(7) can be linearized. The minimax form present in objective function (2) can be linearized by introducing a single continuous variable v to be minimized, and simultaneously requiring v to upper bound the formation time for every selected provider in region $r \in \mathcal{R}$, stage $s \in \mathcal{S}$, and provider $p \in \mathcal{P}_{rs}$.

Moreover, the max function in the left-hand side of constraint (6) can also be linearized through disjunctive programming techniques by introducing additional auxiliary variables and constraints. Specifically, for each $s \in \mathcal{S}$ introduce a continuous variable y_s, as well as binary variables d_{rsp} for each region $r \in \mathcal{R}$, stage $s \in \mathcal{S}$, and provider $p \in \mathcal{P}_{rs}$, for a total of $\sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} |\mathcal{P}_{rs}|$ binary variables.

For each stage $s \in \mathcal{S}$, define constants $\bar{t}_s = \max_{rp}\{t_{rsp}\}$, and let $y_s = \max_{rp}\{t_{rsp}x_{rsp}\}$. These latter equality restrictions will be enforced implicitly via the following three constraint sets:

$$y_s \geq t_{rsp}x_{rsp}, \qquad \forall\ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \tag{8}$$

$$y_s \leq t_{rsp}x_{rsp} + \bar{t}_s\left(1 - d_{rsp}\right), \qquad \forall\ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \tag{9}$$

$$\sum_{r \in \mathcal{R}} \sum_{p \in \mathcal{P}_{rs}} d_{rsp} = 1, \qquad \forall\ s \in \mathcal{S}. \tag{10}$$

Proposition 1 Constraint sets (8)–(10) implicitly ensure $y_s = \max_{rp}\{t_{rsp}x_{rsp}\}$ for all $s \in \mathcal{S}$.

Proof. The equality needs to hold for all stages $s \in \mathcal{S}$. Thus, consider any $\tilde{s} \in \mathcal{S}$. Constraint set (4) ensures at least one $x_{r\tilde{s}p} = 1$ as we can assume $\sum_{r \in \mathcal{R}} D_{r\tilde{s}} > 0$. Constraint set (8) implies that $y_{\tilde{s}}$

is at least as large as the largest $t_{r\tilde{s}p}$ value for which $x_{r\tilde{s}p} = 1$, hence $\max_{rp}\{t_{r\tilde{s}p}x_{r\tilde{s}p}\} \leq y_{\tilde{s}}$. Consider the two terms in the right-hand side of constraint set (9). The latter term $\bar{t}_{\tilde{s}}(1 - d_{r\tilde{s}p})$ acts as a switch that allows for positive slack in the amount of $\bar{t}_{\tilde{s}}$ (enough for any assignment in the first term) to assist in satisfying the constraint for all but one $d_{r\tilde{s}p}$ value, as specified in (10). Hence, the constraints (9) for $\tilde{s}$ are satisfied, and can only be satisfied, when $d_{r\tilde{s}p} = 1$ for the indices r and p for which both $t_{r\tilde{s}p}$ takes a maximum value and $x_{r\tilde{s}p} = 1$, i.e., when positive slack is unnecessary. For all other r and p index values, the extra slack of $\bar{t}_{\tilde{s}}$ ensures feasibility. This shows $y_{\tilde{s}} \leq \max_{rp}\{t_{r\tilde{s}p}x_{r\tilde{s}p}\}$. Such an assignment simultaneously ensures feasibility for constraint sets (8)–(10) and also that $y_{\tilde{s}} = \max_{rp}\{t_{r\tilde{s}p}x_{r\tilde{s}p}\}$. The proof is now complete as the choice of $\tilde{s}$ was arbitrary. ■

Hence, we can replace nonlinear constraint set (6) with the following linear constraint in conjunction with (8)–(10) above:

$$\sum_{s \in \mathcal{S}} y_s \leq T. \tag{11}$$

The final, linearized mixed-integer formulation for forging a virtual enterprise with minimized maximum formation time is:

$$\mathit{Minimize} \quad v \tag{12}$$

$$\mathit{subject\ to} \quad v \geq f_{rsp}x_{rsp}, \qquad \forall\ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \tag{13}$$

$$\sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{P}_{rs}} c_{rsp}x_{rsp} \leq B, \tag{14}$$

$$\sum_{p \in \mathcal{P}_{rs}} a_{rsp} x_{rsp} \geq D_{rs}, \qquad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, \tag{15}$$

$$\sum_{p \in \mathcal{P}_{rs}} (q_{rsp} - Q_{rs}) x_{rsp} \geq 0, \qquad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, \tag{16}$$

$$\sum_{s \in \mathcal{S}} y_s \leq T, \tag{17}$$

$$y_s \geq t_{rsp} x_{rsp}, \qquad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \tag{18}$$

$$y_s \leq t_{rsp} x_{rsp} + \bar{t}_s (1 - d_{rsp}), \qquad \forall \ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}, \tag{19}$$

$$\sum_{r \in \mathcal{R}} \sum_{p \in \mathcal{P}_{rs}} d_{rsp} = 1, \qquad \forall \ s \in \mathcal{S}, \tag{20}$$

$$v \in \mathbb{R}; \ y_s \in \mathbb{R}, \forall \ s \in \mathcal{S}; \ d_{rsp}, x_{rsp} \in \{0,1\}, \forall \ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}. \tag{21}$$

Formulation (12)–(21) is a mixed (binary) integer *linear* program that is equivalent to (2)–(7), containing $|\mathcal{S}|$ +1 continuous variables, $2 \cdot [\sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} |\mathcal{P}_{rs}|]$ binary variables, and $3 \cdot [\sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} |\mathcal{P}_{rs}|] + 2 \cdot |\mathcal{R}||\mathcal{S}| + |\mathcal{S}| + 2$ constraints. While larger in size, this is advantageous because (12)–(21) can now be passed directly to the state-of-the-art mixed-integer programming solvers such as CPLEX (IBM ILOG CPLEX, 2018). We refer to the formulation (12)–(21) as $\text{AVRSC}_{\text{OPT}}$, which can be used to find the optimal assignment of providers to regions and stages that minimizes the maximum time to formation while respecting constraints on budget, quality, and cycle time[2].

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[2] At optimality, y_s^* by construction assumes the exact value of the maximum cycle time for each stage $s \in \mathcal{S}$. This is likely very valuable for practical analysis purposes, such as for examining the largest per-stage cycle times, and especially when comparing multiple high-quality solutions. If this precision in

We note that there are some significant advantages to modeling provider decisions with binary variables, including that the same provider can appear in multiple stages and/or regions, simply by adding side constraints enforcing such relationships. Further, if minimum levels of activity, implications, disjunctions, and other logical conditions exist, they could also be readily added in our proposed formulation (12)–(21).

Methodology for Generating Diverse, High-Quality Solutions

Having linearized the $\text{AVRSC}_{\text{OPT}}$ formulation, we can now use an off-the-shelf optimization solver to identify a single optimal solution (assuming one exists and can be found). However, given the uncertain planning environment surrounding the formation of virtual reverse supply chains, it may be of interest to have a collection of high-quality solutions, and particularly so if they are structurally distinct from one another. It is the identification of high-quality and yet diverse solutions to $\text{AVRSC}_{\text{OPT}}$ that we now address.

Rationale for Portfolios of High-Quality and Diverse Solutions

A portfolio of diverse, high-quality solutions can empower managers with greater flexibility in their decision-making, as the additional solutions can be used to weigh subtle decision factors that are not easily modeled. Some of these uncertainties may arise from intangible factors and

y_s^* is not necessary, then constraint sets (19), (20), and the binary d variables may be omitted without impacting the correctness of the model with respect to the optimal assignment decisions x^* and minimized maximum formation time v^*.

managerial preferences or perceptions that may be problematic to quantify. For example, there may be situations in which a decision-maker has a historical perspective or prior experience regarding certain service providers (e.g., some may be poorer collaborators) that would be difficult to express analytically. While the standard output of an optimization solver, being only a single solution, does not offer such flexibility, a portfolio of diverse, high-quality solutions can offer more alternatives with respect to provider selection, and thereby provides the rationale for such portfolios.

Novelty of Present Work with Respect to High-Quality and Diverse Solution Methodology

For the sake of brevity, we outline the solution methodology to generate a set of diverse optimal and near-optimal solutions in Appendix A in the Online Companion. For complete details on the methodology, we refer the interested reader to Trapp and Konrad (2015).

We remark on two novel contributions that the present work makes with respect to this methodology. Specifically, this is the first demonstration of the technique to problems beyond 0-1 integer optimization problems, namely to *mixed* 0-1 integer problems, by evaluating diversity precisely over the structural binary variables, that is, the 0-1 x^*_{rsp} values concerning which providers to select. Because the other d^*_{rsp}, y^*_s and v^* variables are auxiliary and used for bookkeeping, their interpretation contributes nothing meaningful with respect to diversity. Hence, we ensure the integrity of the computed diversity by considering only the structural decision variables. Second, in the present work there are some additional considerations necessary to normalize the two objective terms of diversity and quality, which we separately demonstrate using disjunctive programming techniques in Appendix B in the Online Companion.

Evaluating Collective Diversity

Let $\boldsymbol{X}$ be the set of solutions generated using the methodology. The collective diversity of the solutions in $\boldsymbol{X}$ can be assessed using the $D_{bin}(\boldsymbol{X})$ metric (31) (Danna and Woodruff 2009), where n represents the dimension of the x vector in any solution:

$$D_{bin}(\boldsymbol{X}) = \frac{2}{n|\boldsymbol{X}|(|\boldsymbol{X}|-1)} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{p\in\mathcal{P}_{rs}} \sum_{j=1}^{|\boldsymbol{X}|-1} \sum_{h=j+1}^{|\boldsymbol{X}|} \left| x_{rsp}^{(j)*} - x_{rsp}^{(h)*} \right|. \tag{22}$$

.

The $D_{bin}(\boldsymbol{X})$ metric takes a value between 0 and 1, and provides the average (scaled) pairwise distance between all solutions in $\boldsymbol{X}$. Larger values of $D_{bin}(\boldsymbol{X})$ indicate portfolios having greater collective diversity. This is because the inner absolute value term, which takes values of 0 or 1, causes the $D_{bin}(\boldsymbol{X})$ expression to increase when it evaluates to 1, indicating a difference between variable values between two solutions $x_{rsp}^{(j)*}$ and $x_{rsp}^{(h)*}$.

Illustrative Example

To further motivate the impending computational study, we provide an illustrative example on a small test instance with two regions, three stages, and three or four providers in each region-stage. We characterize four types of solutions to illustrate the contribution of our methodology. Specifically, the four solutions are: an optimal solution (i.e., $\boldsymbol{x}^{(0)^*}$); a high-quality and relatively diverse solution from $\boldsymbol{x}^{(0)^*}$ – as generated by the aforementioned methodology; a solution that is high-quality, but not very diverse from $\boldsymbol{x}^{(0)^*}$; and finally, a solution that is diverse, but not of high quality. To keep the example manageable, we depict each of the four solutions featuring the capacity that selected providers can make available; however, we leave out from the figure additional complicating details such as meeting budget, quality, and cycle time restrictions to keep the depiction from being too busy. Highlighting the available capacity of each solution is sufficient to characterize the four types of solutions. These four solutions are depicted in Figure 3.

The first row in Figure 3 shows an optimal solution $\boldsymbol{x}^{(0)^*}$ that happens to have a minimized maximum formation time of 40 days (though this metric is not visually displayed), while the fourth row contrasts a second solution that is quite diverse from $\boldsymbol{x}^{(0)^*}$ – in fact, greater than 70% pairwise diversity, as measured by (22). However, its quality lags behind, as the formation time is 44 days. The third row depicts a solution that shares with $\boldsymbol{x}^{(0)^*}$ the same minimized maximum formation time of 40 days; however, when comparing to top row, it can be seen that its solution structure is not very diverse from that of $\boldsymbol{x}^{(0)^*}$. The second row presents a solution that is both high in quality (indeed, it is another optimal solution, having formation time of 40 days), and yet diverse from $\boldsymbol{x}^{(0)^*}$; in fact, expression (22) exceeds 35% pairwise diversity with respect to $\boldsymbol{x}^{(0)^*}$.

	Region 1											Region 2										
	Stage 1: Collection				**Stage 2: Pre-Processing**				**Stage 3: Refurbishing**			**Stage 1: Collection**				**Stage 2: Pre-Processing**				**Stage 3: Refurbishing**		
	P1	**P2**	**P3**	**P4**	**P1**	**P2**	**P3**	**P4**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**	**P4**	**P1**	**P2**	**P3**	**P4**	**P1**	**P2**	**P3**
Optimal solution	49	0	0	51	57	43	0	0	50	0	50	48	0	52	0	0	0	53	47	0	53	47
High-quality and diverse	49	0	0	51	48	0	52	0	50	0	50	43	57	0	0	35	0	65	0	52	48	0
High-quality, less diverse	49	0	0	51	57	43	0	0	50	0	50	48	0	52	0	35	0	65	0	52	48	0
Diverse, not high-quality	0	51	49	0	31	0	34	35	0	47	53	0	40	33	27	0	39	0	61	55	0	45

Figure 3: Visualizing solutions for a small test instance with two regions (first tier / row), three stages (second tier / rows), and multiple providers (third tier / row). Four types of solutions are visualized in the subsequent rows: optimal with minimized maximum formation time of $\boldsymbol{v}^* = \mathbf{40}$ in the first row, high-quality and diverse from optimal with $\boldsymbol{v}^* = \mathbf{40}$ in the second row; high-quality with $\boldsymbol{v}^* = \mathbf{40}$, but not diverse from optimal in the third row; and not high-quality with $\boldsymbol{v}^* = \mathbf{44}$, but diverse from optimal in the fourth row. Each cell denotes the percentage of capacity made available by a provider (e.g. P1) for a particular region and stage, where darker shadings indicate greater capacity made available. By comparing rows (in particular, the first and second rows), the existence of high-quality solutions that are also diverse becomes clear.

Computational Experiments

It is useful to investigate the behavior of the methodology to find multiple high-quality and diverse solutions to the optimization model under varying conditions. We propose test classes for $\text{AVRSC}_{\text{OPT}}$ in a mobile phone context and subsequently discuss the computational performance of the methodology.

Computational Setup

We propose three test classes. The number of stages, $|\mathcal{S}|$, is likely to be rather static, and we take it to be $|\mathcal{S}| \in \{2,3,4,5\}$ in all experiments. Similarly, for any given region r and stage s, we take the number of potential providers $|\mathcal{P}_{rs}| \in \{3,4,5\}$. The three test classes fundamentally differ in the number of regions $\mathcal{R}$; we separate them into small, medium, and large. The small test class features $\mathcal{R} \in \{2,3,4,5\}$, the medium class has $\mathcal{R} \in \{6,7,8,9,10\}$, while the large test class has $\mathcal{R} \in \{[10,50] \cap \mathbb{Z}\}$, so that the classes experience increasing number of regions from small, to medium, to large. For each of the three test classes we randomly generated 1,000 test instances. The largest of these was a test instance of $\mathcal{R} = 50$, $S = 5$, and over 1,000 total potential providers, leading to a formulation (12)–(21) with more than 2,000 binary variables and 3,500 constraints.

Other key parameters include formation time f_{rsp}, which was set to between 15 and 45 days for each provider, cycle time t_{rsp}, taken to be between 20 and 36 days for each stage, and demand D_{rs}, which after some careful calculations was taken to be in the range of 654 and 1,308 kilograms available for collection per day, or between 19,620 and 39,240 per month[3,4]. All of the parameters were independently generated and uniformly distributed over their respective domains.

Computational Environment

Our approach was coded in C++ and compiled using g++ version 4.4.7 20120313 (Red Hat 4.4.7-4) using 2 Intel(R) Xeon(R) E5-2690 CPUs each with 8 cores running at 2.90GHz and 64GB RAM. All optimization was performed using the callable library of IBM ILOG CPLEX 12.5.1 (IBM ILOG CPLEX, 2019). We set a one-hour limit to solve any optimization problem, and prioritized numerical stability by setting the CPX_NUMERICAL_EMPHASIS parameter to CPX_ON. We set $P = 10$ to return, where possible, ten diverse, high-quality solutions. Even so, most individual problems solved in seconds.

Summary of Computational Results

Table 2 summarizes key performance metrics on the 1,000 test instances for each of the three classes. In particular, the algorithm returned a set of $\boldsymbol{X}$ solutions, $0 \leq |\boldsymbol{X}| \leq 10$, for all 3,000 instances. The second column of Table 2 details the number of instances that returned ($|\boldsymbol{X}| =$) 10 solutions, while the other four columns provide, for each respective class, mean values for four measures *over those instances that returned a full* $|\boldsymbol{X}|$*=10 solutions*. Standard deviations are

[3] This information is derived from actual industry sources for cell phones. Canalys. Smart phones overtake client PCs in 2011. Available from: https://www.canalys.com/newsroom/smart-phones-overtake-client-pcs-2011 [cited 2019 July 5]

[4] Environmental Protection Agency. Electronics Waste Management in the United States through 2009; May 2011. Available from https://nepis.epa.gov/Exe/ZyPURL.cgi?Dockey=P100BKKL.TXT [cited 2019 July 5]

recorded in parentheses, and mean runtimes are measured in CPU time.

Table 2: Summary Performance Metrics of Solved $AVRSC_{OPT}$ Instances, by Test Classes

Test Class	**Count of $\lvert X \rvert$=10**	**Mean Runtime To Find X (s)**	**Mean Iterations**	**Mean Gap**	**Mean $D_{bin}(X)$**
Small	770	50.74 (83.90)	1.88 (0.33)	0.09 (0.09)	0.44 (0.09)
Medium	385	33.01 (72.01)	1.78 (0.41)	0.05 (0.04)	0.43 (0.09)
Large	99	92.34 (148.58)	1.59 (0.49)	0.02 (0.03)	0.48 (0.06)

The small test class featured the least number of infeasible test instances (1,000 – 770 = 230); as the randomly generated test instances grew in size, so did the tendency for the resulting models to be infeasible. Over all test classes, the average runtime to generate a portfolio of 10 solutions was 48.6 seconds, which is a relatively modest amount of time in our estimation. Even for the largest of test classes, the algorithmic runtime was in our opinion well within the time needed to make a critical, yet strategic, decision on constructing a reverse virtual supply chain. Over all test classes and instances, the maximum runtime for any instance was 624.6 seconds, amounting to about 10 minutes of CPU time. These short runtimes were directly related to the low iteration counts (on average, less than two) of the implementation of Dinkelbach's algorithm, referenced further in Appendix A in the Online Companion, which is known to have super-linear convergence properties (Schaible, 1976). This bodes well if the model were to be extended in the future; while likely larger in size, it may still be amenable to finding solutions via off-the-shelf mixed-integer programming solvers.

Discussions on Quality and Diversity of Solutions to $AVRSC_{OPT}$

We now address the performance of the methodology with respect to the diversity and quality of generated solutions.

Analysis of Solution Diversity

Recall that the collective diversity calculated in (22) and reported in Table 2 is measured over precisely the x^*_{rsp} components of all solutions in $\boldsymbol{X}$, as they are the only variables over which diversity is meaningful. Table 2 displays some rather high observations for mean diversity metrics across all three test classes, specifically with respect to other studies (Trapp and Konrad 2015; Petit and Trapp 2015; Petit and Trapp 2019; Trapp and Sarkis 2016). This means that the solutions in the set $\boldsymbol{X}$ are quite diverse, and specifically there is significant flexibility in choice of suitable providers to meet the specified demand in all region-stage pairs. Looking across the three test classes individually, there does not appear to be a significant trend towards more or less diversity as the test instances grow in size from small to medium to large. This implies that there is adequate flexibility inherent in the model with respect to the choice of providers.

Analysis of Solution Quality

The quality of any solution x can be quantified by determining its gap from optimality (i.e., distance from the minimum objective function value z^*). The optimality gap is expressed in the following manner, where ϵ is taken to be a very small but positive number:

$$\frac{v - z^* + \epsilon}{z^* + \epsilon}, \tag{23}$$

where v represents the *minimized* maximum formation time as per $AVRSC_{OPT}$. In all cases, the mean optimality gap was within 10% of the global optimal solution, indicating the solutions were of high quality, especially in light of the fact that, as in many real-life scenarios, many of the data appearing in formulation (12)–(21) are likely to come from best-guess approximations. Across the three test classes, the mean optimality gap appears to be largest in the small test class, where a mean gap of 0.09 was observed, with the medium test class coming in at 0.05, and the large at 0.02.

Thus, there is a clear decreasing trend in the mean optimality gap as the test class sizes grow; equivalently, it can be observed that the quality of the solution set $\boldsymbol{X}$ improves with the size of the test class. For $AVRSC_{OPT}$, this is likely due to the objective function and its relation to the randomly generated values for the $\boldsymbol{f_{rsp}}$ parameter. For small test instances, there are at most 25 region-stage pairs, allowing for a greater fluctuation in the minimum possible maximum formation time. For test instances in the largest class, however, there can be up to 250 region-stage pairs; for such an instance it is very unlikely to find an optimal objective function value much below the maximum possible value of the $\boldsymbol{f_{rsp}}$ parameter, hence the gaps for alternate solutions are likely to be smaller.

Figure 4, Figure 5, and Figure 6 depict the mean optimality gap distribution as expressed in (23) for each instance, for the small, medium, and large test classes, respectively. In each figure, the majority of the solutions are in the first two bins that have the smallest optimality gap (i.e., those that are closer to optimal), thus demonstrating that most solutions tend to be high-quality. Hence, they are another representation of the high quality of the obtained diverse solution sets.

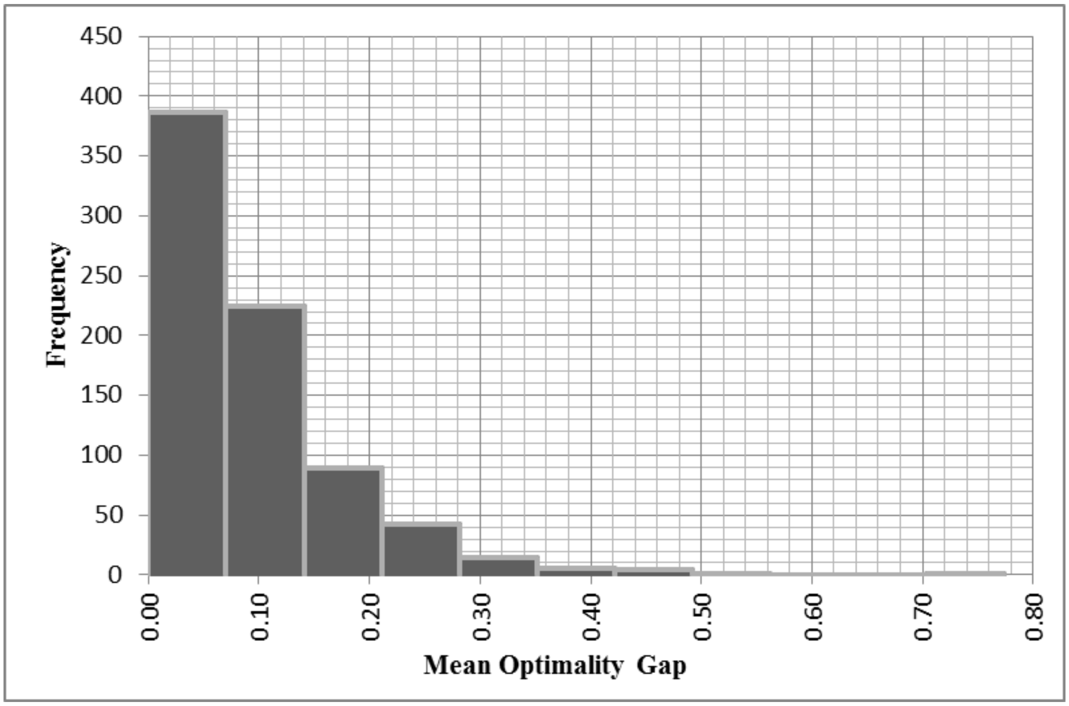

Figure 4: Histogram of Mean Optimality Gap in Solution Sets: Small Test Class

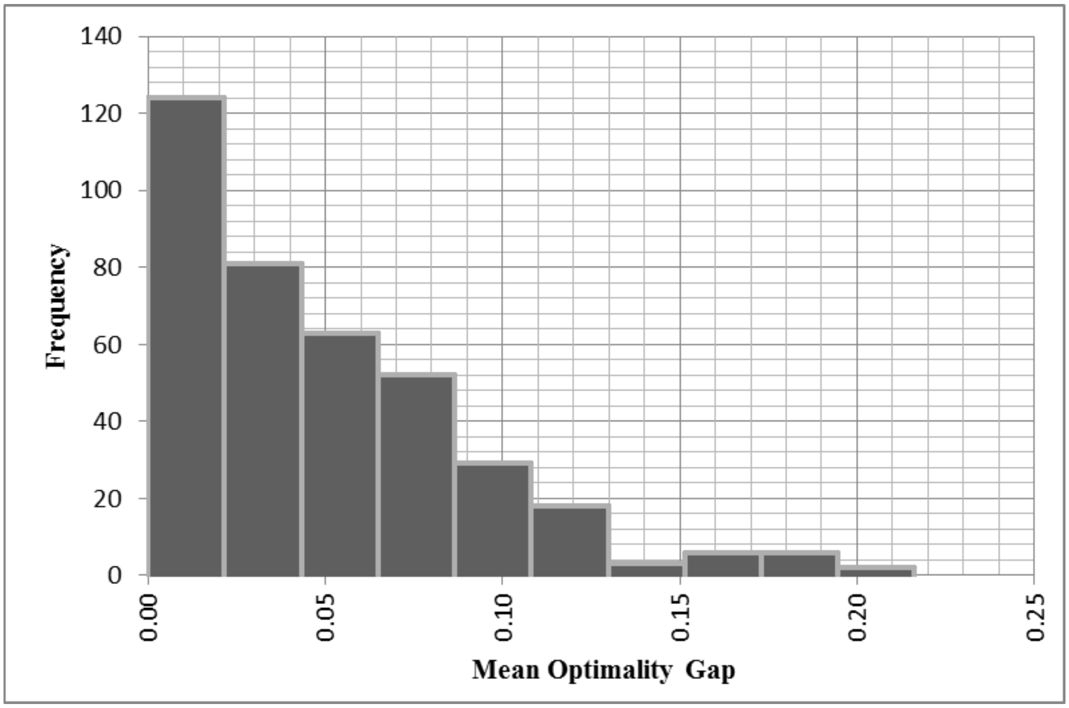

Figure 5: Histogram of Mean Optimality Gap in Solution Sets: Medium Test Class

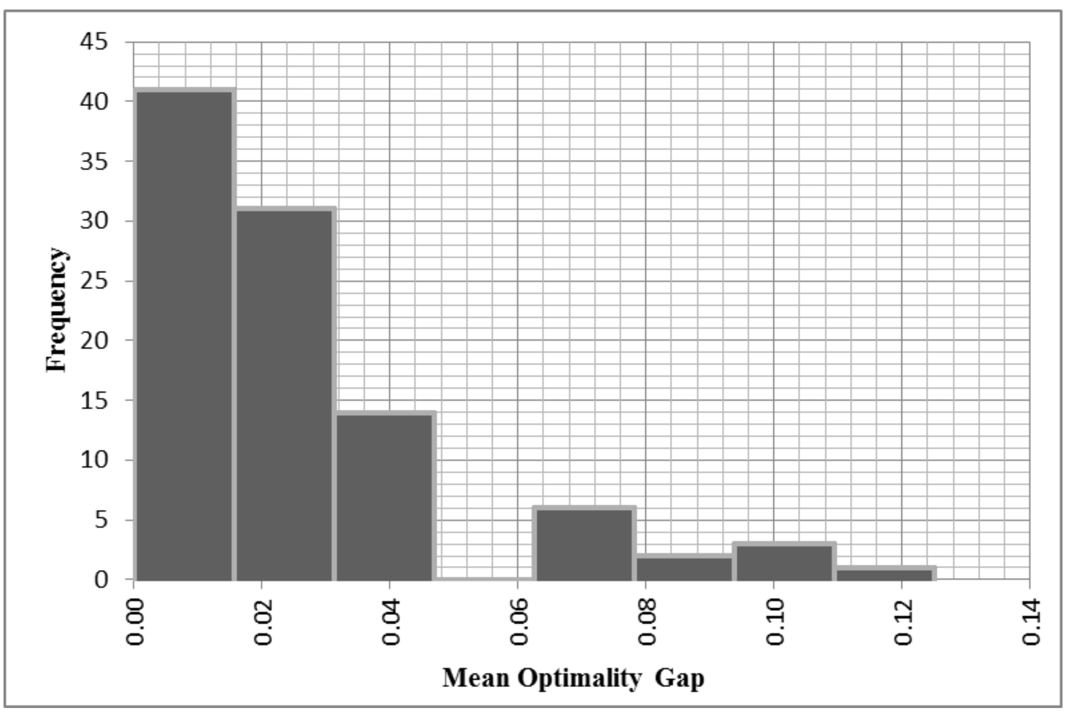

Figure 6: Histogram of Mean Optimality Gap in Solution Sets: Large Test Class

Discussions and Conclusions

Given the numerous sustainability and business concerns facing organizations and their product streams, the issue of "closing-the-loop" in supply chains has received increased attention over the past two decades (Zhu et al., 2008). Critical to closing the loop is the effective use of reverse supply chains. Because many organizations do not have the necessary capabilities to effectively and efficiently manage the entire reverse supply chain, turning to fourth party service providers for expertise and resources may be a prudent strategy. Market opportunities and sensitivities can drive the need for short-term, product or material-specific reverse supply chains. In such cases, a rapid formation of an agile virtual enterprise for reverse supply chain activities is essential, and was the main motivation for the present study. The models and decision tools introduced in this paper can prove valuable to OEMs and others whose products need to be managed in an extended producer responsibility or related context. The techniques put forth both offer practical value, and also contribute to the research of diverse optimal and near-optimal solutions.

We developed a mathematical programming representation for the problem of composing an agile virtual enterprise in the context of end-of-use mobile phones. The model has explicitly incorporated a variety of infrastructure and performance characteristics. For each region and stage in the reverse supply chain process, the formulation seeks to choose a set of providers that can meet the specified demand and quality, and time limitations, with the aim of minimizing the maximum formation time for the entire virtual reverse supply chain. While the original integer formulation is nonlinear, we introduce two linearizations that make it amenable to state-of-the-art mixed-integer linear programming solvers.

Practical and Managerial Implications

There are considerable challenges and uncertainties inherent in assembling an agile virtual enterprise. To this end, we discussed how to obtain a portfolio of optimal and near-optimal solutions that explicitly incorporate diversity, so as to provide flexibility in managerial decision-making. This flexibility is particularly useful in situations such as forming a virtual enterprise, where there exists intangible and unquantifiable dynamics that affect decision-making. This technique improves computational results by providing a variety of high-quality solutions, while addressing concerns related to the judgmental aspects of many parameters and data that is acquired.

Managers, organizations and decision makers can parlay the solutions to determine the most effective partnerships from a diverse set of solutions. For example, there may be some difficulty in determining and incorporating intangible factors such as some specific partner combinations being historically less than effective collaborators. Our approach provides managers the opportunity to use intuition and informal knowledge to identify the ideal high quality solution. Thus providing a portfolio of such solutions yields a more flexible and realistic opportunity to identify an effective solution, given political, social, and other intangible factors.

While we considered mobile phone technology, our approach is applicable to other situations that require rapid reverse supply chain formation, are time-sensitive, and need to consider intangible factors. Construction management, product recalls, and natural disaster response are perhaps other areas that could benefit from the techniques proposed. Using the results from our approach, managers in these and similar situations can effectively parlay tangible and intangible factors for evaluation of the partnership formation system. As with any model or set of considerations, the necessary data needs to be developed, stored and acquired, for these models to prove effective. Setting up a user-friendly decision support system to show alternatives effectively, will be an important future step for additional managerial buy-in.

Managers tend to adopt and apply solutions that they understand – a critical aspect of managerial buy-in and acceptance (Churchman, 1964). While the “high-quality” and “diverse” solution terminology has distinct specific mathematical definitions that can vary based on the practical settings, the practical philosophy and approach is intuitive. We also believe that our approach can provide greater managerial acceptance because, by providing multiple solutions, it allows for managerial discernment to play a role (Camm 2014). The diverse solution set may also provide managers a starting point for discussion for strengths and weaknesses, even political considerations, of the various comparative sets (Siebert and Keeney 2015). Such discussion may allow for identification of hidden factors that may need to be explicitly included in the optimization model, as well as any sequential decision models that can more effectively incorporate intangible factors into the managerial decision-making environment.

Research Implications

Moreover, the present context is the first application to adapt the solution technique of Trapp and Konrad (2015) to a *mixed* binary integer linear program, i.e. one that contains continuous decision variables. The insight that the approach is applicable to cases where formulation transformations involve only auxiliary continuous variables is an important contribution to the developing diverse optimal and near-optimal solution paradigm (Trapp and Konrad 2015; Petit and Trapp 2015; Petit and Trapp 2019; Trapp and Sarkis 2016). Our computational results demonstrate that, for moderately-sized problems, portfolios of solutions can be quickly generated that feature diverse optimal and near-optimal solutions.

Although the contributions of this work are both practical and theoretical, some limitations exist for the application and model. At the same time, these limitations provide opportunities for further development and research. First, the modeling effort included only a single product with a single objective. Extending this effort to include multiple products, multiple time periods, and alternate or multiple objectives (e.g. cost), are all possibilities for future research directions. Although experimentation with realistic data and problem scope were incorporated, actual data and implementation in a real-world setting would be beneficial. Issues in real-world settings such as data incompleteness, the feasibility of acquiring all necessary information, and acceptance by management may be limitations of the model and its practical application, and should be further investigated. Finally, the solution set is a portfolio of high-quality and diverse solutions for a given set of constraints and performance metrics. There may be additional managerial, perceptual and intangible data (e.g. trust, reputation, legitimacy, relationships) that are not easy to model within the context of an optimization program. To overcome some of these limitations, linkage of the outcomes from the model introduced here to other multiple criteria decision approaches, such as AHP/ANP that can incorporate a broader set of intangible measures and decision factors, can enhance the methodology.

Supply chain management researchers have ample opportunities to help develop analytical models to solve emergent concerns of various economic, sustainable, social, and regulatory pressures. This paper builds not only on the foundations for virtual enterprises and reverse supply chains, but also provides insight into additional research avenues where some of these pressing issues can be more fully understood and addressed.

Appendix A: Details on Methodology to Generate Diverse, High-Quality Solutions

In an effort to be self-contained, we discuss additional details on the methodology we use to generate a diverse set of high-quality solutions to $AVRSC_{OPT}$. Some of this material is necessarily similar in nature to Trapp and Konrad (2015) and Trapp and Sarkis (2016).

Given two binary vectors of the same dimension, diversity between the two can be viewed as the sum of the vector indices for which the corresponding values are not in agreement. For example, the diversity between vector A = [0 0 1 0] and vector B = [1 1 1 0] is 2. This measure can be expressed, for example, using the L_1 (Manhattan, or taxicab) distance, and it naturally extends to a collective diversity measure when there are more than two binary vectors, as will be discussed below. In general, however, it can be challenging to obtain a portfolio of solutions that are both high in quality and yet collectively diverse. There are two primary reasons. First, solutions that score relatively high in quality tend to evaluate poorly with respect to diversity, due to structural similarities. Second, diverse solutions typically come from disparate areas of the feasible region, and do not likely share similar quality characteristics. When both high quality and diversity are emphasized, this situation produces conflict.

Let us denote by S the set of constraints (13)-(20) together with the variable domains expressed in (21). For the sake of exposition, assume $AVRSC_{OPT}$ has at least one feasible solution, and an optimal solution vector exists: $x^* = (v^*, y_s^*, d_{rsp}^*, x_{rsp}^* \; \forall \, r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P})$, with optimal objective function value z^*. Let $\boldsymbol{X}$ denote the set of all presently identified solutions, so that initially $\boldsymbol{X} = \{x^*\}$, and consider the following fractional objective:

$$R(x) = \frac{Relative\ Solution\ Diversity}{Relative\ Deterioration\ in\ Objective\ Quality}. \quad (24)$$

Objective (24) expresses the ratio of the relative solution diversity to the relative deterioration in the objective function quality, namely the minimized maximum formation time. Assume any other feasible solution exists $\bar{x}$ with maximum formation time $\bar{v}$. The denominator representing the deterioration in objective quality can be obtained with $\bar{v} - z^* + \epsilon$ (where ϵ is a small positive value, ensuring the denominator takes a nonzero value in the event $\bar{v} = z^*$). The diversity of $\bar{x}$ with respect to the elements in $\boldsymbol{X}$ can similarly be represented and will be discussed below.

To find a single high-quality solution that is diverse from x^*, we replace the objective of $\text{AVRSC}_{\text{OPT}}$ with fractional objective (24), to be maximized, referring to the resulting formulation as M-$\text{AVRSC}_{\text{OPT}}$. Although (24) is nonlinear, it is possible to solve this class of optimization problem, namely a nonlinear fractional binary integer program, via Dinkelbach's algorithm (Dinkelbach, 1967). This approach solves a sequence of linearized problems related to the original nonlinear fractional programming problem. Any solutions in $\boldsymbol{X}$, for example x^*, can be prevented from being revisited using linear expressions that forbid the binary variable values (Balas and Jeroslow, 1972):

$$\sum_{i:x_i^*=0} x_i + \sum_{i:x_i^*=1} (1 - x_i) \geq 1. \quad (25)$$

Then, solving M-$\text{AVRSC}_{\text{OPT}}$ with (25) will generate a solution $x^{(1)^*}$ distinct from $x^* = x^{(o)^*}$ that maximizes the ratio in (24), that is, it simultaneously emphasizes solution quality and diversity (with respect to all elements in $\boldsymbol{X}$), where diversity is computed via the *centroid* *diversity measure*. The centroid is the vector composed of the component-wise average of each element over all solutions in $\boldsymbol{X}$:

$$\boldsymbol{c} = \left(\boldsymbol{c}_i = \frac{1}{h}\sum_{j=0}^{h-1} x_i^{(j)^*}\right). \quad (26)$$

This *centroid diversity metric* computes the distance of any vector $x \in \{0,1\}^n$ from the elements of X in the following fashion:

$$\sum_{i=1}^{n} \boldsymbol{c}_i + \sum_{i=1}^{n}(1 - 2\boldsymbol{c}_i)x_i. \quad (27)$$

Putting all of this together, now consider a sequential process of finding alternate solutions for some iteration $h > 1$. This entire process can be repeated as often as desired, as long as there remain feasible solutions, to generate a solution set $\boldsymbol{X}$. An overview of this process is depicted in Figure 7.

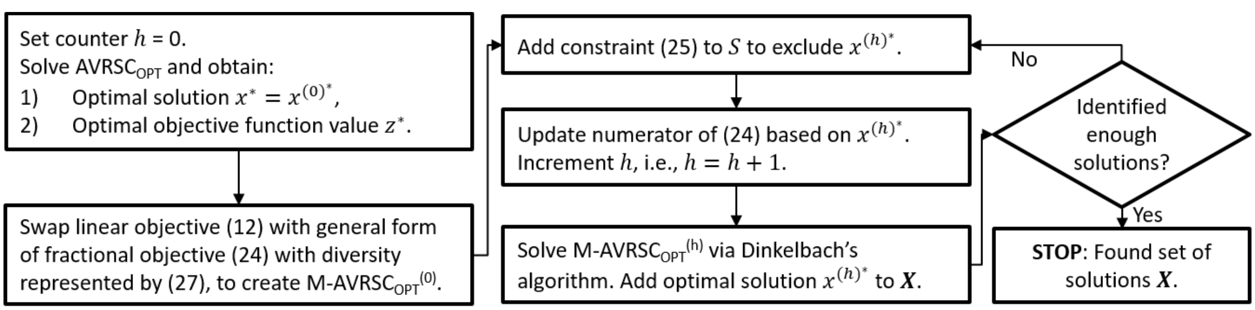

Figure 7: Methodology for Generating Diverse Set of High-Quality Solutions to AVRSC$_{OPT}$.

Additional information on these metrics, as well as an implementation of Dinkelbach's algorithm, are contained in Trapp and Konrad (2015). Details on normalizing the ratio objective function can be found in Appendix B.

Appendix B: Normalizing Objective Function Terms of Diversity and Quality

Determining the normalization factor $\mathcal{F}_h$ for iteration h requires the solution of two additional optimization problems. While the denominator in $\mathcal{F}_h$ is straightforward to compute, the numerator involves knowledge of the *maximum* feasible formation time to assemble the reverse logistics virtual enterprise. This can be accomplished via a *maximax* formulation that modifies formulation (12) – (21) using disjunctive programming techniques. Specifically, the following five steps support the transition:

(1) Add a new continuous variable v, and replace objective function (12) to *maximize* v;

(2) Add $\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}} |\mathcal{P}_{rs}|$ auxiliary binary variables g_{rsp};

(3) Remove constraint set (13);

(4) Add the following set of constraints, where f^{max} and f^{min} are the maximum and minimum formation times, over all r, s, and p, respectively, i.e. $f^{max} = \max_{rsp}\{f_{rsp}\}$ and $f^{min} = \min_{rsp}\{f_{rsp}\}$:

$$v \leq f_{rsp}x_{rsp} + f^{max}(1 - x_{rsp}) + (f^{max} - f^{min})(1 - g_{rsp}), \quad \forall\ r \in \mathcal{R}, s \in \mathcal{S}, p \in \mathcal{P}_{rs}; \tag{28}$$

(5) Add a single, final constraint:

$$\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}\sum_{p\in\mathcal{P}_{rs}} g_{rsp} = 1. \tag{29}$$

These modifications are designed to allow for v to take the *largest* formation time from those x_{rsp} variables for which $x_{rsp} = 1$. The optimal value of v^* is used as the value for the numerator in the normalization factor $\mathcal{F}_h$.

Proposition 2 The maximax reformulation (12)–(21) ensures v takes the value of the maximum formation time among all feasible solutions.

Proof. The new objective seeks to identify the largest formation time v. An obvious upper bound for v is f^{max}, though it is possible that no feasible solution exists with $v = f^{max}$ due to other constraints. For any r, s, and p, there are four cases for the corresponding constraint set (28) depending on the assignments of binary variables x_{rsp} and g_{rsp}. When $x_{rsp} = 0$, constraint set (28) becomes trivially satisfied. When $x_{rsp} = 1$ and $g_{rsp} = 0$, it can be seen that again (28) provides an upper bound no tighter than f^{max}. Now constraint (29) assigns one and only one g_{rsp} variable to unity, and so when both $x_{rsp} = 1$ and $g_{rsp} = 1$, (28) becomes $v \leq f_{rsp}$. Given the modified objective to maximize v, an optimal solution will feature $g_{rsp} = 1$ for indices r, s, and p for which there is no larger f_{rsp} value with $x_{rsp} = 1$ feasible. Otherwise, the objective could strictly improve. This completes the proof. ■

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