WORCESTER POLYTECHNIC INSTITUTE
NINETEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 18, 2006
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the **Individual Answer Sheet** provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators **MAY NOT** be used.

1. A triangle is determined by three straight lines:

$$y = \sqrt{3}\,x + 2$$
$$y = 2$$

and

$$y = mx + b$$

Find a value for m so that the triangle is *equilateral.*

2. If the line $y = mx + 1$ intersects the ellipse $x^2 + 4y^2 = 1$ exactly once then the value of m^2 is equal to what?

3. A circle passes through the vertices of a triangle with side lengths $7\frac{1}{2}$, 10 and $12\frac{1}{2}$. What is the radius of the circle?

4. If an arc of 60 degrees of circle I has the same length as an arc of 45 degrees of circle II then the ratio of the area of circle I to circle II is_____.

5. A fair die is rolled 6 times. What is the probability of rolling at least a 5 at least 5 times?

6. In an equilateral triangle, the area is equal to the perimeter. What is the radius of the circumscribed circle?

7. What is the sum of the series

$$103 + 106 + 109 + 112 + \ldots + 523?$$

8. Factor $x^4-8x^3-9x^2+92x+140$ as completely as possible.

9. A parabola has its focus at the point **(4,0)** and its directrix is the line $y = -4$. A beam of light travels right to left, and parallel to the x axis and strikes the parabola where $x = 8$, reflecting off of it. At what coordinates will the reflected beam contact the parabola again?

10. The number $(2^{48}-1)$ is divisible by two numbers between 60 and 70. What are they?

11. Fermat's Little Theorem states that if **p** is prime and **a** not a multiple of **p** then

$$a^p \bmod p \equiv a$$

If it has been computed that $5^{106483} \bmod 106483 \equiv 6586$ then what can be concluded about the number **106483** from this?

(For two numbers to be *equivalent* or *congruent mod p*, indicated by $\equiv$, their difference must be a multiple of p. Thus $38 \bmod 6 \equiv 14$ because $38 - 14 = 24 = 4(6)$)