WORCESTER POLYTECHNIC INSTITUTE

Eighteenth Annual Invitational Math Meet
October 19, 2005
Individual Exam Question Sheet

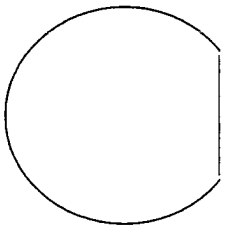
Directions: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

1. What is $\sum_{i=1}^{6} i^2$ modulo 7?

2. Evaluate $\sqrt{(2^3 \cdot 3^2)^6 \div (4^4)^3}$.

3. Given the equation $3x + 5y + 7 = 9$, find the equation of the line perpendicular to the line at $y = 2$.

4. Let $D(x) = x^2 + a^2 - b^2$, where a and b are the next two consecutive integers after integer x. What are the roots of D(x)?

5. 

If a segment of length r, equal to the radius of the circle, truncates the circle, what is the area of the truncated circle in terms of r?

6. What is the sum of $102 + 104 + \cdots + 196 + 198 + 200$?

7. Evaluate x given that $\log_{\cos(\frac{\pi}{6})} \sin(\log_{10}(x)) = 1$

8. What is the smallest positive integer P for which $\mathbf{5760P = N^3}$, where N is an integer?

9. An uncrossed belt is fitted without slack around two circular pulleys with radii of 14" and 4". If the distance between the points of contact of the belt with the pulleys is 24", then what is the distance in inches between the centers of the pulleys?

10. For the quartic equation $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$, the product of two of its four roots equals -32. Determine the value of coefficient k.

11. The number 2^{4096} ends in what digit?