WORCESTER POLYTECHNIC INSTITUTE

SEVENTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 20, 2004
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

[1] What is $\log_{+3}(+243)$?

[2] What are the coordinates of the focus of the parabola whose equation is given by

$$-8y + x^2 - 14x + 25 = 0?$$

[3] Find the coefficient of x^7 in $(2 - x)^9$.

[4] Find a cubic polynomial which has roots at -2, $+5$ and $+2$. Your answer must be in standard form $Ax^3 + Bx^2 + Cx + D$.

[5] Compute $3^{940} \bmod 79$.

[6] Express the (base 10) number 140 as a base 3 number.

[7] If $\sin(x) = 3\cos(x)$ find $\sin(x)\cos(x)$.

[8] If a and b are integers such that $x^2 - x - 1$ is a factor of $ax^3 + bx^2 + 1$ then b has what value?

[9] Find a value for a so that the following system of equations has no solution:

$$\begin{aligned} x + y &\quad - z = 2 \\ x + 2y &+ z = 3 \\ x + y &+ (a^2 - 5)z = a. \end{aligned}$$

[10] A coin is flipped 10 times. How many possible outcomes contain at most three heads?

[11] Assume that x, y and z are all greater than 1 and let w be a positive number such that

$$\log_x w = 24 \qquad \log_y w = 40 \qquad \log_{xyz} w = 12$$

Find $\log_z w$.

NAME Answer Key

SCHOOL

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QUESTION	ANSWER	SCORE
1	$\boxed{5}$	
2	$\boxed{(7,-1)}$	
3	$\boxed{-144}$	
4	$\boxed{x^3-5x^2-4x+20}$ or any scalar or multiple	
	# CORRECT × 1 =	

QUESTION	ANSWER	SCORE
5	$\boxed{2}$	
6	$\boxed{12012_3}$	
7	$\boxed{3/10}$	
8	$\boxed{b=-2}$	
	# CORRECT × 2 =	

QUESTION	ANSWER	SCORE
9	$\boxed{a=-2}$	
10	$\boxed{176}$	
11	$\boxed{60}$	
	# CORRECT × 3 =	

Individual Total