WORCESTER POLYTECHNIC INSTITUTE

THIRTEENTH ANNUAL INVITATIONAL MATH MEET

OCTOBER 26, 2000

TEAM EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators MAY NOT be used.

[1] How many x values satisfy the equation $x\cos(x) - \sin(x) = 0$ for $-8 \leq x \leq +8$?

[2] Express the numbers $22_3 + 161_7$ in binary (base 2). The subscripts (3 and 7) represent the base of each number.

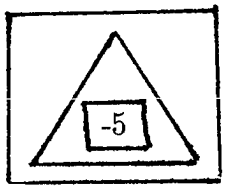
[3] Let $\boxed{n}$ be defined as:

$$\boxed{n} = \begin{cases} n \text{ if } n <= 0 \\ 2n+3 \text{ if } n > 0 \end{cases}$$

let $\triangle n$ be defined as

$$\triangle n = \begin{cases} |n-1| \text{ if } n <= -5 \\ |n^2| \text{ if } -5 < n < 5 \\ 3n-4 \text{ if } n >= 5 \end{cases}$$

What is



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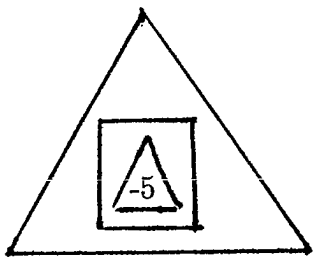
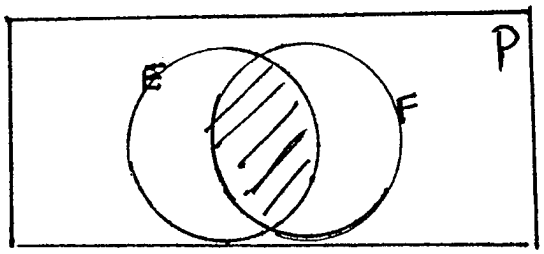


?

4 Which of the following expresses the shaded set shown? Choose **ALL** that apply.



(The notation $\overline{A}$ means the complement of A relative to P)

a) $E \cap F$

b) $E \cup F$

c) $P \cap (E \cap F)$

d) $P \cup (E \cap F)$

e) $\overline{\overline{F} \cup \overline{E}}$

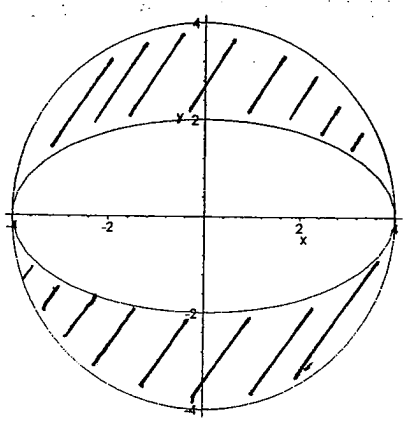
f) $\overline{\overline{F} \cap \overline{E}}$

g) $(\overline{\overline{F} \cup \overline{E}}) \cup P$

h) $(\overline{\overline{F} \cap \overline{E}}) \cap P$

5 What two whole numbers, neither containing any zeros, when multiplied together equal exactly 1,000,000,000?

6 Shown below is an ellipse inscribed in a circle. What is the area of the shaded region?



7 The sum of all but one of the interior angles of a convex polygon equals 700°. What is the value of the remaining angle?

8 In the sequence of $1, 3, 2, -1, \ldots$ each term after the first two is equal to the term preceding it minus the term preceding that. The **sum** of the first one hundred terms of the sequence is equal to what?

9 For what real numbers x is the quantity $\sqrt{x^3 + x^2 - 10x + 8}$ a real number?

10 In the triangle ABC, D is the midpoint of AB, E is the midpoint of DB; and F is the midpoint of BC. If the area of triangle ABC is 96, what is the area of triangle AEF?

11 Evaluate

$$\log_{10}(\tan 1^\circ) + \log_{10}(\tan 2^\circ) + \log_{10}(\tan 3^\circ) + \ldots + \log_{10}(\tan 88^\circ) + \log_{10}(\tan 89^\circ)$$

12 If a and b are positive integers and

$$(2^{ab} - 1) = (X)(2^{ab-a} + 2^{ab-2a} + 2^{ab-3a} + \cdots + 2^a + 1)$$

What is X?

13 How many points with **integer** coordinates does the graph of

$$3x + 7y = 1$$

pass through?

14 A road which is level passes 12′ above another level road at right angles to it. A bicyclist rides along the lower road at a speed of 9 feet per second. At the exact instant the bicycle passes underneath the upper road, a car passes overhead going four times as fast as the bicycle. One second later, how far apart are they?