WORCESTER POLYTECHNIC INSTITUTE

TWELFTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 21, 1999
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

[1] For what real values of x does the following identity hold?

$$\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}=2\sqrt{x-1}$$

[2] Solve for t so that

$$\frac{9^{n+2}+81^{2n}27^{1-2n}}{\sqrt[3]{8^2 3^{6n}}}=3^t$$

[3] Alice says she has twice as many brothers as sisters, but her brother has twice as many sisters as he has brothers. How many sisters does Alice have?

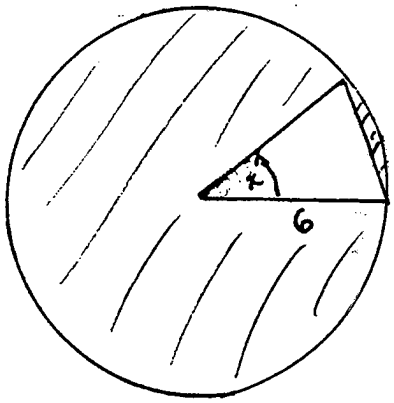
[4] Find the smallest positive integer in the set

$$\{96x-270y \mid x, y \text{ are integers}\}$$

5 If $\tan(\alpha) + \cot(\alpha) = m$, express $\tan^4(\alpha) + \cot^4(\alpha)$ in terms of m.

6 Consider the parabola $y = \frac{1}{8}x^2$. A ray comes down vertically, parallel to the y axis, and reflects off the parabola at the point whose x coordinate is 3. What is the x coordinate of the point where it strikes the parabola a second time?

7 In the following diagram, which is **not** drawn to scale, the shaded area is 32π. What is $\sin(x)$?



8 Consider a triangle in the coordinate plane with vertices $(1, 1), (2, 3)$ and $(x, 2)$ where $x > 1$. What is the area of the triangle?

9 Let $a = \underbrace{1111\ldots1111}_{1998\,\text{digits}}$ and $b = \underbrace{2222\ldots2222}_{999\ \text{digits}}$. Compute $\sqrt{a-b}$

10 If a baseball team has a probability P of beating its opponent in any one game, independently of what happens in any other games, what is the probability it wins a 5 game series? (a winner is the first team to win 3 games)

11 Consider a circle with n points on its boundary. How many possible points of intersection are there between the chords inside the circle? Assume that no three chords intersect at the same point.