WORCESTER POLYTECHNIC INSTITUTE

Eleventh Annual Invitational Math Meet : October 22, 1998

INDIVIDUAL EXAM : questions & Solutions

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators **MAY NOT** be used.

[1] **If six times the larger of two real numbers is four times the smaller number, and the larger number is five more than the smaller number, find the sum of the two numbers.**

-25

[2] **For positive integers n, it is known that $1+3+5+\cdots+(2n-1)=n^2$.**

Evaluate $\dfrac{1+3+5+\cdots+739}{741+743+\cdots+1479}$

$\frac{1}{3}$

[3] **Given a triangle with sides having lengths $4\sqrt{3}$, 12, and $8\sqrt{3}$. Find the length of the bisector of the second largest angle.**

8

[4] **In a class of 50 students, 32 are males, 41 are right-handed and 26 are right-handed males. How many left-handed females are in the class?**

3

[5] **A standard six-faced die is rolled. Independently, a coin is tossed once. What is the probability that either the die shows a six or the coin shows a head (but not both)?**

$\frac{1}{2}$

6 Assume that the earth and moon are perfect spheres and that the ratio of the volume of the earth to the volume of the moon equals 64 to 1. What is the ratio of the surface area of the earth to the surface area of the moon?

16 to 1

7 How many permutations of the digits $0, 1, \ldots, 9$ have an odd digit in the first position and $1, 2, 3, 4$ or 5 in the last position?

887,040

8 In a triangle with sides of length a, b and c, $(a+b+c)(a-b-c)+bc=0$. Find the measure, in degrees, of the angle θ opposite the side of length a.

120

9 A floor of infinite width and length is constructed of square tiles of sides $1, 2$ and 4 in inches. The numbers of these tiles are, respectively, in the ratios $24 : 2 : 1$. If a pin is tossed on the floor at random, what is the probability that the tip of the pin lands in a square of side 2 inches? (Assume that the probability that the tip of the pin lands on a line separating squares is zero.)

$$\frac{1}{6}$$

10 In the hexadecimal number system (base 16), we use the symbols 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F. The symbols A-F are interpreted as decimal 10-15. Convert the decimal number 20,385 to hexadecimal.

4FA1

11 A cow is tied with a 30 foot rope to an outside corner of a rectangular barn having dimensions 10 feet by 20 feet. How many square feet of grazing area does the cow have?

800π