WORCESTER POLYTECHNIC INSTITUTE

Tenth ANNUAL INVITATIONAL MATH MEET

OCTOBER 23, 1997

INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS : Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

1 Determine the number of distinct prime factors of 39039.

2 If $x^a x^b = 1$ and $x \neq \pm 1$, evaluate $4a - b^2 + a^2 + 4b - 10$.

3 In a geometric sequence of real numbers, the sum of the first two terms is 7 while the sum of the first six terms is 147. What is the sum of the first four terms?

4 The 1967 World Almanac indicated that Rhode Island had a population of 892,709 with a population density (population/sq. mile) of 843.8, while Worcester County in Massachusetts had a population of 583,228 with a population density of 384.7. Which is larger in area, Worcester County or Rhode Island?

5 Mina received \$200 to spend in Paris. The exchange rate is 5.88 French francs to the dollar. There is a 4.5% tax included in the prices in France. She knows, however, that being a U.S. citizen, she will not have to pay the tax. What is the highest price (in French francs) of an article that Mina can buy?

6 Each student in a college must go for lottery to obtain housing. One out of every five applicants (selected at random) is offered housing on campus. If a student enters the lottery for two consecutive years, what are her chances of getting an offer for housing?

7 What is the 1997th digit in the decimal representation of the number 1/13?

8 A polynomial has remainder 8 when divided by $x-2$, remainder 3 when divided by $x-3$, and remainder -6 when divided by $x-4$. Find the remainder when the same polynomial is divided by $(x-2)(x-3)(x-4)$.

9 A function f, defined for all non-zero real numbers x, satisfies $3f(x) + 4f(1/x) = 5x$. Find all x for which $f(x) - f(-x) = 0$.

10 If an arc of 45^o on Circle I has three times the length of an arc of of θ^o on Circle II, and the area of Circle I is four times the area of Circle II, find θ (in degrees).

11 Four circles of the same size are arranged as shown in the adjoining figure. If the total area of the four circles is 36π, what is the diameter of the largest circle that can be drawn in the enclosed (shaded) region?