MA540/4631 **Fall 2012**

HOMEWORK ASSIGNMENTS

Homework #7 **Assigned: 11/27/12** **Due: 12/11/12**

1. (a) Show that if $\sum_{n=1}^{\infty} P(| X_n - X |> \epsilon) < \infty$ for every $\epsilon > 0$, then $X_n \stackrel{a.s.}{\rightarrow} X$.

(b) Let $\{X_n\}$ be a sequence of independent Beta$(1, 2)$, random variable let $X_{(1)} = \min(X_1, \ldots, X_n)$ and $X_{(n)} = \max(X_1, \ldots, X_n)$. Show that $X_{(1)}$ converges almost surely to 0 and $X_{(n)}$ converges almost surely to 1.

2. Casella and Berger, Problem 5.42.

3. (a) Let $X_n \stackrel{d}{\rightarrow} X$ and $Y_n \stackrel{p}{\rightarrow} a$, where a is a constant. Show that $X_n + Y_n \stackrel{d}{\rightarrow} X + a$.

(b) Let $X_n \stackrel{p}{\rightarrow} X$. Let $g(x)$ be any continuous function and let $Y_n = g(X_n)$ be another sequence. Show that $Y_n \stackrel{p}{\rightarrow} Y$, where $Y = g(X)$.

(c) If $\sqrt{n}(Y_n - \mu) \stackrel{d}{\rightarrow}$ Normal$(0, \sigma^2)$, then $Y_n \stackrel{p}{\rightarrow} \mu$. Prove this result directly and by using Stutsky's theorem.

4. (a) Casella and Berger, Problem 5.35.

(b) Casella and Berger, Problem 5.44.

5. Show that for two random variables X and Y with appropriate moments,

$$E(\frac{X}{Y}) \approx \frac{\mu_x}{\mu_y}, \quad \text{and} \quad Var(\frac{X}{Y}) \approx \left(\frac{\mu_x}{\mu_y}\right)^2 \{cv_x^2 + cv_y^2 - 2\rho cv_x cv_y\},$$

where $cv_x = \sigma_x/\mu_x$ and ρ is the correlation.

6. Let $f(x)$ be the density function of a random variable X, and suppose that one can draw a sample from $f(x)$ easily. Show how you can use the accept-reject algorithm to come up with a simpler algorithm to draw a sample from $g(x)$ where,

$$g(x) = \frac{1}{1+x^2} f(x) / \int_{-\infty}^{\infty} \frac{1}{1+x^2} f(x) dx, \qquad -\infty < x < \infty.$$